APPENDIX 7A: Margin period of risk and scaling

Consider an exposure over a certain period such as the margin period of risk (MPoR) defined as τ that linearly decreases to zero. Intuitively, the total exposure will be proportional to (considering either standard deviation or variance):

$$\int_0^\tau \frac{\tau - u}{\tau} du = \tau/2.$$

The exposure is therefore equivalent to the same constant exposure for half of the length of the MPoR.

In the discrete case, the variance of a position that decreases linearly at the end of each day will be proportional to:

$$\left[\left(1-\frac{1}{n}\right)^2 + \left(1-\frac{2}{n}\right)^2 + \cdots + \left(1-\frac{n-1}{n}\right)^2\right] = \frac{n}{3}\left(1-\frac{1}{n}\right)\left(1-\frac{1}{2n}\right).$$

Compared to a constant position, the standard deviation of the linearly amortising position will be smaller by the following multiplier:

$$\sqrt{\frac{1}{3}\left(1-\frac{1}{n}\right)\left(1-\frac{1}{2n}\right)}$$

This multiplier converges to $\sqrt{1/3}$ as n becomes large which is expected since:

$$\int_0^\tau \left(\frac{\tau - u}{\tau}\right)^2 du = \tau/3.$$

Using the discrete formula: for example, a 10-day constant exposure is roughly equivalent to a 5.3 day linearly amortising one (inputting $n = 10$ in the above formula gives a multiplier of 0.5339).