APPENDIX 20A: FX exposure profile calculations

Assume that the FX rate follows a geometric Brownian motion (lognormal distribution). FX forwards can be calculated by:

$$FX_{0,t} = FX_0.\exp\left[\left(r_d - r_f\right).t\right]$$

with r_d and r_f being continuously compounded interest rates and FX_0 being the spot FX rate.

The value of an FX forward of maturity T at some future date $t(< T)$ is:

$$\exp\left[-r_f(T-t)\right].FX_t - \exp[-r_d(T-t)].K$$

with K being the contractual FX rate.

The future FX rate is given by:

$$FX_t = FX_0.\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma.W_t\right]$$

with μ being the drift, σ the volatility and W_t the random Gaussian variable. Since the value of the FX forward will be monotonic with the FX rate then we can calculate the PFE directly from the given move. The move in the FX rate at the quantile α will be:

$$W_t = \Phi^{-1}(\alpha).\sqrt{t}$$

and the corresponding FX rate will therefore be:

$$FX_t^{\alpha} = FX_0.\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma.\Phi^{-1}(\alpha).\sqrt{t}\right]$$

Substituting this in the formula for the value of the FX forward therefore gives the PFE as:

$$PFE_t^{\alpha} = \exp\left[-r_f(T-t)\right].FX_0.\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma.\Phi^{-1}(\alpha).\sqrt{t}\right] - \exp[-r_d(T-t)].K$$

Note that this allows a drift to be input and does not need to be risk-neutral (although this is likely in practice).

Since the EPE and ENE are expectations they cannot be treated similarly but can be evaluated as options leading to Black-Scholes type formulas:

$$EPE(t) = \exp\left[-r_f(T-t)\right].FX_0.\exp(\mu t).\Phi(d_1) - K.\exp[-r_d(T-t)].\Phi(d_2)$$

$$ENE(t) = \exp\left[-r_f(T-t)\right].FX_0.\exp(\mu t).\Phi(-d_1) - K.\exp[-r_d(T-t)].\Phi(-d_2)$$

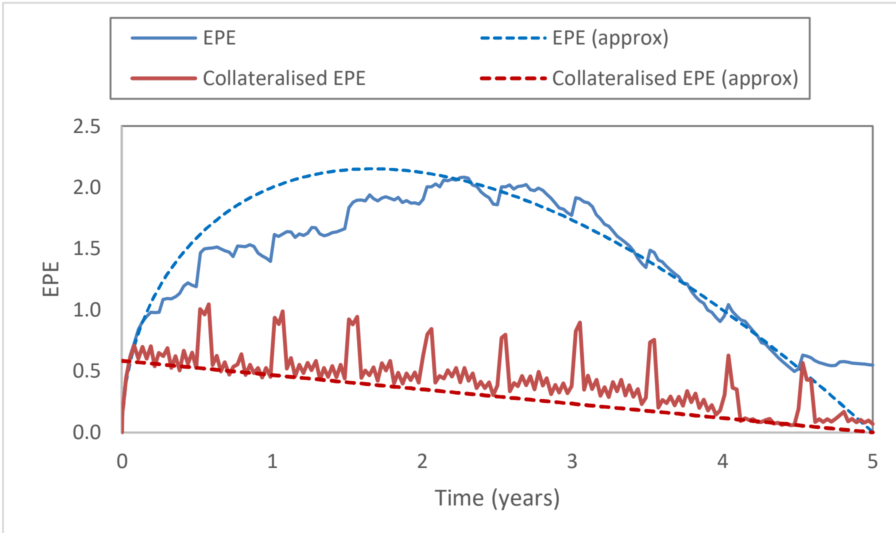
$$d_1 = \frac{\ln\left(\frac{FX_0.\exp(\mu t)}{K.\exp\left[-(r_d - r_f)(T-t)\right]}\right) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

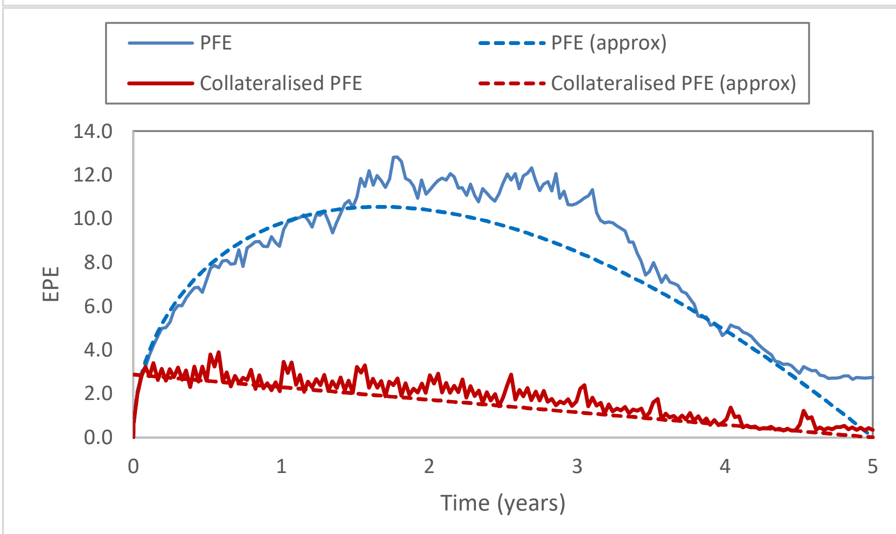
These formulas are implemented in Spreadsheet 20.2.

APPENDIX 20B: Approximation of collateralised EPE and PFE

The Figures below show the exposure profiles (EPE/PFE) in Figure 20.28 in the book and their approximations via the functions described in Appendix 14. These approximations are given below.

	Uncollateralised	Collateralised
EPE	$\sigma\sqrt{t}(T-t)/\sqrt{2\pi}$	$\sigma\sqrt{\tau_{MPoR}}(T-t)/\sqrt{2\pi}$
PFE	$\sigma\sqrt{t}(T-t).\Phi^{-1}(\alpha)$	$\sigma\sqrt{\tau_{MPoR}}(T-t).\Phi^{-1}(\alpha)$





The ratio of the uncollateralised to collateralised EPE and PFE in the above Figures is 3.30 and 4.51 whereas the general formula gives $\frac{8}{15}\sqrt{5 \times 365/20} = 5.09$.