APPENDIX 18A: Average spreads and credit indices

As described in Appendix 11A, a credit default swap (CDS) has two legs corresponding to the premium payments and a contingent default payment (the latter in the event a credit event occurs during the lifetime of the contract).

An index CDS contract involves trading protection on a number of reference entities (e.g. 125) that are defined as being within the specified index. These reference entities are usually equally weighted which means that a unit position in a CDS on a credit index with n reference entities is approximately equivalent to trading an amount of $1/n$ in each of the single-name CDS contracts.

The use of approximately in the above comparison is explained as follows. A buyer of protection on a credit index effectively has protection on each of the reference entities within that index. Using Equation (11.6) in Appendix 11A (and the definitions therein), this can be written as:

$$\sum_{i=1}^{n} w_i LGD \int_0^T \exp(-ru)\, h \exp(-hu)\, du$$

where w_i represents the weight of each individual reference entity (usually $1/n$). The above expression is additive across the individual CDS in the index (the total value of index protection is equal to the total weighted value of individual reference entity protection). On the other hand, the premium payment on a CDS index position will be (Equation 11.5):

$$X_{index} \sum_{i=1}^{n} w_i \int_0^T \exp(-(r + h_i)u)\, du$$

The contractual index spread (X_{index}) is not equivalent to the (weighted) average of the premium payments on the underlying CDS which is:

$$\sum_{i=1}^{n} w_i X_i \int_0^T \exp(-(r + h_i)u)\, du$$

Indeed, due to the fact that the risky annuity term is strictly decreasing function of h_i then:

$$X_{index} < \sum_{i=1}^{n} w_i X_i$$

This occurs because in a portfolio of CDS, when a high spread name defaults then the total spread paid will reduce significantly whereas in an index trade it will only reduce proportionately.

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APPENDIX 18B: Simple formula for super senior LGD

Suppose that the losses on two tranches of debt, junior and super senior, are given by the formulas in Equations 18.5 on the book. If the overall distribution of the loss is uniform then:

$$LGD_{junior} = (1-p)^{-1}\int_0^{1-p} L.dL + \int_{1-p}^{1} (1-p).dL = (1+p)/2$$

$$LGD_{ss} = p^{-1}\int_{1-p}^{1} [L-(1-p)].dL = p/2$$

So, whilst the LGD for the two classes of debt depends on their relative proportions (defined by p which is the relative proportion of super senior debt), the ratio of super senior to junior debt LGD is given by:

$$\frac{LGD_{ss}}{LGD_{junior}} = \frac{p/2}{(1+p)/2} = \frac{p}{(1+p)}$$