APPENDIX 13A: ColVA formula

Define ColVA as being the difference between discounting a set of n cashflows at a given rate minus the valuation using a certain 'base' rate:

$$ColVA = \sum_{i=1}^{n} CF_{t_i} DF_{t_i} \exp[-s_{t_i}.t_i] - \sum_{i=1}^{n} CF_{t_i} DF_{t_i} \quad (1)$$

where CF_{t_i} is the value of the (net) cashflow at time t_i, DF_{t_i} is the base discount factor and s_{t_i} represents a spread between the two rates. The spread could represent a different collateral remuneration rate (for example).

Now define the expected future value (EV) for a time t_j as being the current value of all cashflows after that time in the future and discounted using the base rates:

$$EV(t_j) = \sum_{i=j+1}^{n} CF_{t_i} DF_{t_i} \quad (2)$$

It therefore follows that the value of a net cashflow on a given date can be written via a difference in EVs:

$$CF_{t_i} DF_{t_i} = EV_{t_{i-1}} - EV_{t_i} \quad (3)$$

From substituting Equation (3) into Equation (1) then we obtain:

$$ColVA = \sum_{i=1}^{n} [EV_{t_{i-1}} - EV_{t_i}]\{\exp[-s_{t_i} \times t_i] - 1\}$$

$$= \sum_{i=1}^{n} EV_{t_{i-1}}\{\exp[-s_{t_i}.t_i] - 1\} - \sum_{i=1}^{n} EV_{t_i}\{\exp[-s_{t_i}.t_i] - 1\}$$

$$= \sum_{i=1}^{n} EV_{t_{i-1}} \exp[-s_{t_i}.t_i] - \sum_{i=1}^{n} EV_{t_i} \exp[-s_{t_i}.t_i] - \sum_{i=1}^{n} EV_{t_{i-1}} + \sum_{i=1}^{n} EV_{t_i}$$

$$= \sum_{i=1}^{n} EV_{t_{i-1}} \exp[-s_{t_i}.t_i] - \sum_{i=1}^{n} EV_{t_i} \exp[-s_{t_i}.t_i] - EV_{t_0} + EV_{t_n} \quad (4)$$

Since EV_{t_n} and $t_0 = 0$ and therefore $\exp[-s_{t_0}.t_0] = 1$ we can re-write as:

$$= \sum_{i=1}^{n} EV_{t_{i-1}} \exp[-s_{t_i}.t_i] - \sum_{i=0}^{n-1} EV_{t_i} \exp[-s_{t_i}.t_i]$$

$$= \sum_{i=1}^{n} EV_{t_{i-1}} \exp[-s_{t_i}.t_i] - \sum_{i=1}^{n} EV_{t_{i-1}} \exp[-s_{t_{i-1}}.t_i]$$

$$= -\sum_{i} EV_{t_{i-1}}\{\exp[-s_{t_{i-1}}.t_{i-1}] - \exp[-s_{t_i}.t_i]\} \quad (5)$$

The formula above is defined as ColVA in Equation (13.3) in the book. The term $\{\exp[-s_{t_{i-1}}.t_{i-1}] - \exp[-s_{t_i}.t_i]\}$ can be considered to be a forward spread. This formula is also illustrated in Spreadsheet 13.1.

In the case of a mandatory break, in order to use a ColVA formula (as in Spreadsheet 13.2), we first define the break time as t_k with the discounting change only affecting cashflows after this time. Proceeding along similar lines, we can get to a modified version of Equation (4) which cannot be simplified any further:

$$ColVA = \sum_{i=k}^{n} EV_{t_{i-1}} \exp[-s_{t_i}.t_i] - \sum_{i=k}^{n} EV_{t_i} \exp[-s_{t_i}.t_i] - EV_{t_{k-1}} \quad (4)$$

The term $EV_{t_{k-1}}$ is the expected value at the cashflow payment date (or the last cashflow payment date in the event the break is in between dates)..