APPENDIX 12A: LHP approximation for credit losses

i) The LHP approximation

The large homogeneous pool (LHP) approximation of Vasicek (1997) is based on the assumption of a very large (technically infinitely large) portfolio. The loss distribution is defined via:

$$\Pr(L < \theta) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\theta) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right),$$

where $\Phi^{-1}(.)$ represents a cumulative normal distribution function, PD is the (constant) default probability and ρ the correlation parameter.

ii) The IRB formula details

The Basel II internal rating based (IRB) formula given in Equation (12.1) of the book is based on the above approximation together with the so-called granularity adjustment formula of Gordy (2004). This gives a unexpected default probability which is defined by

$$PD_{99.9\%} = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(99.9\%)}{\sqrt{1-\rho}}\right) - PD,$$

where the functions $\Phi(.)$ and $\Phi^{-1}(.)$ are the standard normal cumulative distribution function and its inverse.

The correlation parameter above, ρ , is linked to the default probability (PD) according to the following equation:

$$\rho = 0.12 \times \frac{1-\exp(-50 \times PD)}{1-\exp(-50)} + 0.24 \times \left(1 - \frac{1-\exp(-50 \times PD)}{1-\exp(-50)}\right)$$

In Equation (13.1) in the book, the factor $MA(PD, M)$ is the maturity adjustment that accounts for potential credit migration and is calculated from PD and M according to:

$$MA(PD, M) = \frac{1 + (M-2.5) \times b(PD)}{1 - 1.5 \times b(PD)},$$

where $b(PD)$ is a function of PD defined as:

$$b(PD) = [0.11852 - 0.05478 \times \ln(PD)]^2.$$

Note that the maturity adjustment is capped at 5 and floored at 1.

APPENDIX 12B: Standardised CVA capital formula

When hedging an asset (1) with another asset (2) then the optimal hedge amount, as defined by the variance minimising strategy, is (see Appendix 19A):

$$\rho_{12}\frac{\sigma_1}{\sigma_2}$$

Not surprisingly, the optimal amount is proportional to the correlation, ρ_{12}, between the assets and the ratio of the standard deviations, σ_1 and σ_2. As would be expected, as the correlation reduces, then the optimal hedge amount does also.

In the BA-CVA formula (BCBS 2017), we would expect a similar effect with respect to single-name and index CDS hedges which have supervisory correlations with respect to the counterparty they are hedging. However, the proxy single-name CDS hedges (i.e. where the correlation with the counterparty is not unity) behave in a strange way and this incentivises overhedging.

i) BA-CVA formula

Using the same notation as BCBS (2023b) and as discussed in Section 12.3.2 of the book document and excluding for brevity the correlation parameter within the formula for the single-name hedges and the hedging misalignment term:

$$SCVA_c = \frac{1}{\alpha}.RW_c.\sum_{NS} M_{NS}.EAD_{NS}.DF_{NS}$$

$$SNH_c = \sum_{h\in c} RW_h.M_h^{SN}.B_h^{SN}.DF_h^{SN}$$

$$IH = \sum_i RW_i.M_i^{ind}.B_i^{ind}.DF_i^{ind}$$

The BCBS formula can be derived from the following assumptions:

- An index hedge is a single global index driven by a Gaussian variable V_{ind}.
- Each counterparty exposure is correlated to the global index by $V_c = \rho V_{ind} + \sqrt{1-\rho^2}V_c'$ with V_c' (and therefore V_c) being standard Gaussian variables and ρ being a correlation parameter.
- Each single name hedge is represented by $V_{SNH} = r_{hc}V_c + \sqrt{1-r_{hc}^2}V_{SNH}' = r_{hc}\left(\rho V_{ind} + \sqrt{1-\rho^2}V_c'\right) + \sqrt{1-r_{hc}^2}V_{SNH}'$ with V_{SNH}' being a standard Gaussian variables.

The variance can be seen to be:

$$\left(\rho\sum_c (SCVA_c - r_{hc}SNH_c) - IH\right)^2 + (1-\rho^2)\sum_c (SCVA_c - r_{hc}SNH_c)^2 + \sum_c (1-r_{hc}^2)SNH_c^2$$

This is the same as the BCBS (2023b) formula without the square root and with the change in definition that the correlation parameter for single-name hedges, r_{hc}, is shown explicitly outside the definition of SNH_c. The choice of parameters representing the hedges is shown below.

	Index	SNH	Counterparty
Index	100%	$\rho . r_{hc}$ $= 50\%, 40\%, 25\%$	ρ
SNH		r_{hc}^2 $= 100\%, 64\%, 25\%$	r_{hc} $= 100\%, 80\%, 50\%$
Counterparty			ρ^2

The final term in the formula is residual volatility from single-name hedges rather than being residual CVA volatility as might be expected. In this representation, the CVA is hedging the single-name CDS position rather than the other way around.

As n increases then the first (systematic) term will dominate and so the optimal hedge will tend (assuming no index hedges) towards $SNH_c = SCVA/r_{hc}$ and will therefore **increase** with decreasing correlation. This incentivises overhedging which is a side-effect of the formula being badly formulated.

See Gregory (2019) for further discussion.