APPENDIX 16A: ColVA formula

Define ColVA as being the difference between discounting a set of n cashflows at a given rate minus the valuation using a certain 'base' rate:

$$ColVA = \sum_{i=1}^{n} CF_{t_i} DF_{t_i} \exp[-s_{t_i}. t_i] - \sum_{i=1}^{n} CF_{t_i} DF_{t_i} \quad (1)$$

where CF_{t_i} is the value of the (net) cashflow at time t_i, DF_{t_i} is the base discount factor and s_{t_i} represents a spread between the two rates. The spread could represent a different collateral remuneration rate, for example.

Now define the expected future value (EFV) as being the value of cashflows after a certain time in the future and discounted using the base rates:

$$EFV(t_j) = \sum_{i=j+1}^{n} CF_{t_i} DF_{t_i} \quad (2)$$

It therefore follows that the value of a net cashflow on a given date can be written via a difference in EFVs:

$$CF_{t_i} DF_{t_i} = EFV_{t_{i-1}} - EFV_{t_i} \quad (3)$$

From substituting Equation (3) into Equation (1) then we obtain:

$$ColVA = \sum_{i=1}^{n} [EFV_{t_{i-1}} - EFV_{t_i}]\{\exp[-s_{t_i} \times t_i] - 1\} \quad (4)$$

Which can be rearranged to give:

$$ColVA = -\sum_{i=1}^{n} EFV_{t_{i-1}} \{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\} \quad (5)$$

where the term $\{\exp[-s_{t_{i-1}} \times t_{i-1}] - \exp[-s_{t_i} \times t_i]\}$ can be considered to be a forward spread. In a perfectly collateralised transaction then the (base rate) discounted expected collateral balance will be equal to the EFV which leads to Equation (16.3) in the book.

The above formulas are also implemented in an example in Spreadsheet 16.1.