A Note on the behaviour of single-name proxy CDS hedges in the BA-CVA formula

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Introduction

When hedging an asset (1) with another asset (2) then the optimal hedge amount, as defined by the variance minimising strategy, is:

$$\rho_{12}\frac{\sigma_1}{\sigma_2}$$

Not surprisingly, the optimal amount is proportional to the correlation, ρ_{12}, between the assets and the ratio of the standard deviations, σ_1 and σ_2.

As would be expected, as the correlation reduces, then the optimal hedge amount does also.

In the BA-CVA formula (BCBS 2017), we would expect a similar effect with respect to single-name and index CDS hedges which have supervisory correlations with respect to the counterparty they are hedging. However, the proxy single-name CDS hedges (i.e. where the correlation with the counterparty is not unity) behave in a strange way and this incentivises overhedging.

BA-CVA formula

Using the same notation as the BCBS (2017) document with the exception of not including the correlation parameter within the formula for the single-name hedges and ignoring the hedging misalignment term:

$$SCVA_c = \frac{1}{\alpha}.RW_c.\sum_{NS} M_{NS}.EAD_{NS}.DF_{NS}$$

$$SNH_c = \sum_{h\in c} RW_h.M_h^{SN}.B_h^{SN}.DF_h^{SN}$$

$$IH = \sum_{i} RW_i.M_i^{ind}.B_i^{ind}.DF_i^{ind}$$

i) BCBS formula

The BCBS formula can be seen to be driven by the following structure:

Index: V_{ind}

Counterparty: $V_c = \rho V_{ind} + \sqrt{1-\rho^2}V_c'$

SNH: $V_{SNH} = r_{hc}V_c + \sqrt{1-r_{hc}^2}V_{SNH}' = r_{hc}\left(\rho V_{ind} + \sqrt{1-\rho^2}V_c'\right) + \sqrt{1-r_{hc}^2}V_{SNH}'$

Where V_{ind}, V_c, V_c', and V_{SNH}' are all independent Gaussian variables

The variance can be seen to be:

$$\left(\rho\sum_c (SCVA_c - r_{hc}SNH_c) - IH\right)^2 + (1-\rho^2)\sum_c (SCVA_c - r_{hc}SNH_c)^2 + \sum_c (1-r_{hc}^2)SNH_c^2$$

This is the same as the BCBS formula without the square root and with the change in definition that the correlation parameter for single-name hedges, r_{hc}, is shown explicitly outside the definition of SNH_c.

The final term in the formula is residual volatility from single-name hedges rather than being residual CVA volatility as might be expected. In this representation, the CVA is hedging the single-name CDS position rather than the other way around.

As n increases then the first (systematic) term will dominate and so the optimal hedge will tend (assuming no index hedges) towards $SNH_c = SCVA/r_{hc}$ and will therefore **increase** with decreasing correlation. This incentivises overhedging.

ii) Alternative formula

An alternative structure would be:

Index: V_{ind}

SNH: $V_{SNH} = \frac{\rho}{r_{hc}} V_{ind} + \sqrt{1-\frac{\rho^2}{r_{hc}^2}} V'_{SNH}$

Counterparty: $V_c = r_{hc}V_{SNH} + \sqrt{1-r_{hc}^2}V'_c = \left(\rho V_{ind} + r_{hc}\sqrt{1-\frac{\rho^2}{r_{hc}^2}}V'_{SNH}\right) + \sqrt{1-r_{hc}^2}V'_c$

The variance is:

$$\left(\sum_c \rho\left(SCVA_c - \frac{1}{r_{hc}}.SNH_c\right) - IH\right)^2 + \sum_c \left(1-\frac{\rho^2}{r_{hc}^2}\right)(r_{hc}SCVA_c - SNH_c)^2 + \sum_c (1-r_{hc}^2)SCVA_c$$

Here it can be seen via inspection that the optimal hedge will be $SNH_c = r_{hc}.SCVA_c$ which is natural. Notice that the formula also contains a final term that can be interpreted as the residual (unhedged) CVA volatility. In this representation, the CVA is being hedged by single-name CDS (rather than the other way around).

When $r_{hc} = 100\%$ or $n = 1$ then two formulas are the same. This is therefore only relevant for single-name proxy hedges where either $r_{hc} = 80\%$ (legal relationship) or $r_{hc} = 50\%$ (same sector and region).

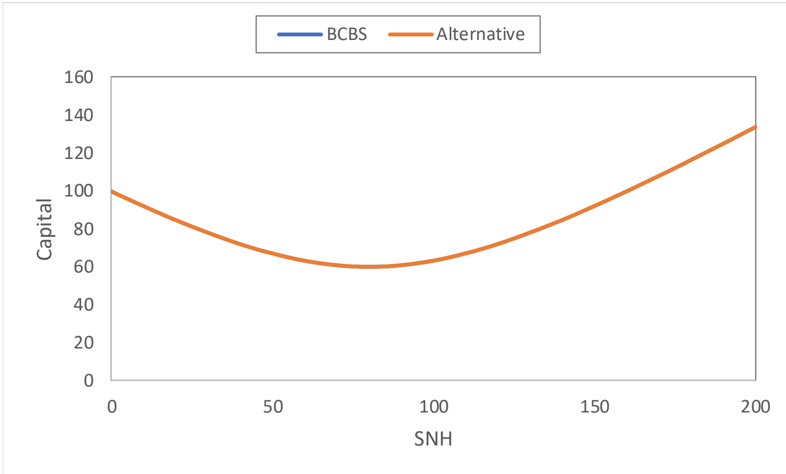
Examples

The results below show the standard deviation (square root of the above formulas) for a homogenous portfolio and parameters of:

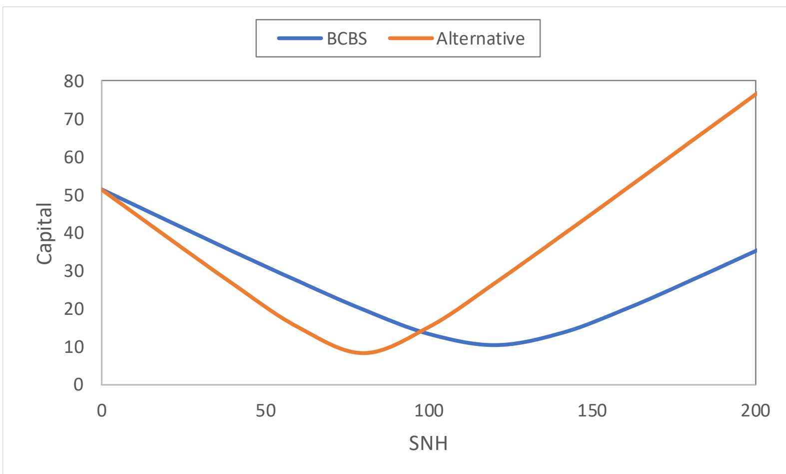
$SCVA = 100$

$\rho = 50\%$

With a single counterparty, the formulas give identical results ($r_{hc} = 80\%$):



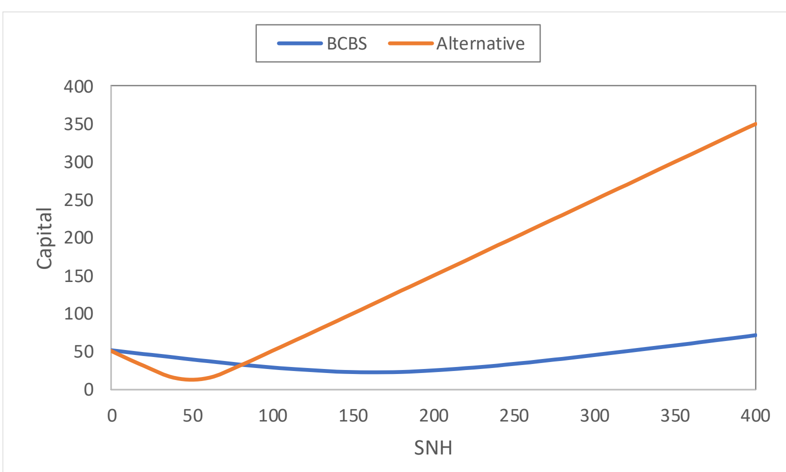
With $n = 50$ and $r_{hc} = 80\%$ (capital per counterparty is shown)



In the above, the BCBS formula has an optimal hedge of 125 ($SCVA/r_{hc} = 100/80\%$) compared to the more obvious 80 ($SCVA \times r_{hc} = 100 \times 80\%$).

Even hedging with **double** the SCVA amount (double the delta neutral hedge) leads to less capital than not hedging.

With $n = 50$ and $r_{hc} = 50\%$



In the above, the BCBS formula has an optimal hedge of approximately 200 ($SCVA/r_{hc} = 100/50\%$) compared to the more obvious 50 ($SCVA \times r_{hc} = 100 \times 50\%$).

Even hedging with **triple** the SCVA amount leads to less capital than not hedging.

References

Basel Committee on Banking Supervision (BCBS), 2015, "Review of the Credit Valuation Adjustment Risk Framework", consultative document, July, www.bis.org

Basel Committee on Banking Supervision (BCBS), 2017, "Basel III: Finalising post-crisis reforms", consultative document, December, www.bis.org