APPENDIX 7A: Simple formulas for PFE, EE and EPE of a normal distribution

Consider a normal distribution with mean μ (expected future value) and standard deviation (of the future value) σ. The future value of the portfolio in question (for an arbitrary time horizon) is given by:

$$V = \mu + \sigma Z,$$

where Z is a standard normal variable.

i) Potential future exposure (PFE)

This measure is similar to that used in value-at-risk (VAR) calculations. The PFE at a given confidence level α, PFE_α, tells us an exposure that will be exceeded with a probability of no more than $1 - \alpha$. For a normal distribution, it is defined by a point a certain number of standard deviations away from the mean: -

$$PFE_\alpha = \mu + \sigma \Phi^{-1}(\alpha),$$

where $\Phi^{-1}(.)$ represents the inverse of a cumulative normal distribution function (this is the function NORMSINV(.) in Microsoft Excel™). For example, with a confidence level of $\alpha = 99\%$, we have $\Phi^{-1}(99\%) = 2.33$ and the worse case exposure is 2.33 standard deviations above the expected future value (defined here by μ).

ii) Expected exposure (EE)

The definition of exposure is:

$$E = \max(V, 0) = \max(\mu + \sigma Z, 0).$$

The EE defines the expected value over the positive future values and is therefore:

$$EE = \int_{-\mu/\sigma}^{\infty} (\mu + \sigma x)\varphi(x)dx = \mu\Phi(\mu/\sigma) + \sigma\varphi(\mu/\sigma),$$

where $\varphi(.)$ represents a normal distribution function and $\Phi(.)$ represents the cumulative normal distribution function. We see that EE depends on both the mean and the standard deviation; as the standard deviation increases so will the EE. In the special case of μ=0, we obtain: $EE_0 = \sigma\varphi(0) = \sigma/2\pi \approx 0.4\sigma$.

The above formulas are used in Spreadsheet 7.1.

iii) Expected positive exposure

The above analysis is valid only for a single point in time. Suppose we are looking at the whole profile of exposure defined by $V(t) = \sigma\sqrt{t}Z$ where σ represents the annual standard deviation (volatility). The EPE, integrating over time, would be defined by:

$$EPE \times T = \varphi(0) \int_0^T \sigma\sqrt{t}dt = \frac{1}{\sqrt{2\pi}}\sigma\frac{2}{3}T^{3/2}$$

Leading to:

$$EPE = \frac{2}{3\sqrt{2\pi}}\sigma\sqrt{T}$$

APPENDIX 7B: Simple representation of exposure profiles

i) Forward

Suppose we want to calculate the exposure on a forward contract and assume the following model for the evolution of the future value of the contract:

$$dV(t) = \mu dt + \sigma dZ,$$

where μ represents a drift and σ is a volatility of the exposure with dZ representing a standard Brownian motion. Under such assumptions the future value at a given time t in the future will follow a normal distribution with known mean and standard deviation:

$$V(t) \sim N\left(\mu t, \sigma\sqrt{t}\right).$$

We therefore have simple expressions for the PFE and EE following from the formulas in Appendix 7A.

$$PFE_t^{\alpha} = \mu t + \sigma\sqrt{t}\Phi^{-1}(\alpha).$$

$$EE_t = \mu t \Phi\left(\frac{\mu}{\sigma}\sqrt{t}\right) + \sigma\sqrt{t}\varphi\left(\frac{\mu}{\sigma}\sqrt{t}\right).$$

ii) Swap

Following the above example and assuming zero drift, an approximation to a swap contract is to assume that the future value at a given time is normally distributed according to:

$$V(t) \sim N\left(0, \sigma\sqrt{t}(T-t)\right).$$

where the $(T-t)$ factor corresponds to the approximate duration of the swap of maturity T at time t. This assumes that the expected future value is zero at all future dates which in practice is the case for a flat yield curve (interest rates the same for all maturities). We can show that the maximum exposure is at $s = T / 3$ by differentiating the volatility term:

$$\frac{d}{dt}\left(\sqrt{t_{max}}(T - t_{max})\right) = \frac{1}{2\sqrt{t_{max}}}(T - t_{max}) - \sqrt{t_{max}} = 0.$$

$$t_{max} = T/3.$$

APPENDIX 7C: Example exposure calculation for a cross-currency swap

Combined the results in the two previous Appendices, we consider a cross currency swap to be a combination of the approximate FX forward and interest rate swap positions.

The FX forward future value follows $N\left(0, \sigma_{FX}\sqrt{t}\right)$ and the interest rate swap follows $N\left(0, \sigma_{IR}\sqrt{t}(T-t)\right)$. Assuming a correlation of ρ between future value of each, the cross-currency swap future value will be given by:

$$V(t) \sim N\left(0, \sqrt{\sigma_{FX}^2 t + \sigma_{IR}^2 t(T-t)^2 + 2\rho\sigma_{FX}\sigma_{IR} t(T-t)}\right).$$

which is used to compute the PFE shown in Spreadsheet 7.3 using the result in Appendix 7A.

APPENDIX 7D: Simple netting calculation

As discussed in Appendix 7A that the EE of a normally distributed random variable is:

$$EE_i = \mu_i \Phi(\mu_i/\sigma_i) + \sigma_i \varphi(\mu_i/\sigma_i).$$

Consider a series of independent normal variables representing transactions within a netting set (NS). They will have a mean and standard deviation given by:

$$\mu_{NS} = \sum_{i=1}^{n} \mu_i$$

$$\sigma_{NS}^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \rho_{ij}\sigma_i\sigma_j$$

where ρ_{ij} is the correlation between the future values. Assuming normal variables with zero mean and equal standard deviations, $\bar{\sigma}$, and correlations, $\bar{\rho}$, the overall mean and standard deviation are given by:

$$\mu_{NS} = 0 \qquad \sigma_{NS}^2 = (n + n(n-1))\bar{\rho})\bar{\sigma}^2$$

Hence, since $\varphi(0) = 1/\sqrt{2\pi}$, the overall EE will be:

$$EE_{NS} = \bar{\sigma}\sqrt{n + n(n-1)\bar{\rho})}/\sqrt{2\pi}.$$

The sum of the individual EEs gives the result in the case of no netting (NN):

$$EE_{NS} = \bar{\sigma}n/\sqrt{2\pi}.$$

Hence the netting benefit will be:

$$\frac{EE_{NS}}{EE_{NN}} = \sqrt{\frac{1 + (n-1)\bar{\rho})}{n}}$$

In the case of perfect positive correlation, $\bar{\rho} = 100\%$, we obtain:

$$\frac{EE_{NS}}{EE_{NN}} = \sqrt{\frac{1 + (n-1))}{n}} = 100\%$$

The maximum negative correlation is bounded by $\bar{\rho} \geq -1/(n-1)$ and we therefore obtain:

$$\frac{EE_{NS}}{EE_{NN}} = \sqrt{\frac{1 - \left(n-1)/(n-1)\right)}{n}} = 0\%$$

