The Impact of Initial Margin

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18 July 2016

Since the global financial crisis, two significant regulatory initiatives have been introduced in the form of the central clearing mandate and bilateral margin requirements. A major implication of these initiatives is the need for major players in the OTC derivative market to post collateral, in the form of initial margin, against their transactions. In this paper, we discuss some of the problems associated with initial posting, such as the quantification of the residual counterparty risk and associated capital requirements. We also characterise a wealth transfer mechanism when parties post initial margin which causes OTC derivative creditors to become more senior at the expense of other creditors. Initial margin has clear funding implications in the form of margin value adjustment (MVA) and the pricing of the underlying wealth transfer effect in unsecured lending transactions.

Keywords: Initial margin, collateral, MVA, wealth transfer, SIMM, central counterparty, BCBS-IOSCO

JEL Classification: C60, G12, G32

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1. Introduction

The global financial crisis from 2007 onwards triggered grave concerns regarding counterparty risk, catalysed by events such as the Lehman Brothers bankruptcy, the failure of monoline insurers and the default of Icelandic banks. Counterparty risk in OTC derivatives, especially credit derivatives, was identified as a major systemic risk to the financial system. One result of this was the implementation of regulation requiring that all standardised OTC derivatives be cleared via central counterparties (CCPs).

Related to the clearing mandate, incoming regulation requires that major OTC derivatives users are subject to bilateral margin rules (BCBS-IOSCO 2015) governing the mechanics by which they post collateral to one another. These rules, which phase in starting from September 2016, basically mean that financial counterparties will need to post variation margin (with zero thresholds) and initial margin to each other. The intention seems to be to make the bilateral collateral practices close to those in centrally cleared markets. The main cost associated to both central clearing and the bilateral margin rules is the up-front posting of this initial margin. Note that, despite its name, initial margin is generally dynamic and will change according to the portfolio in question and market conditions.

It is not surprising to see the cost of initial margin posting being calculated by banks. This component is often known as MVA (margin value adjustment). As a representation of funding costs, MVA has similarities with funding value adjustment (FVA) which has already received a reasonable amount of attention in the literature (for example, see Burgard and Kjaer 2011). In particular, there has been a debate regarding whether or not funding costs should be included in pricing and valuation which was first raised by Hull and White (2012). Despite such views, FVA seems to be commonly quoted by banks in prices with clients and is reported as an adjustment in the financial statements of many banks. For example:

> "*The Firm implemented a Funding Valuation Adjustments ("FVA") framework this quarter for its OTC derivatives and structured notes, reflecting an industry migration towards incorporating the cost or benefit of unsecured funding into valuations. For the first time this quarter, we were able to clearly observe the existence of funding costs in market clearing levels. As a result, the Firm recorded a $1.5B loss this quarter.*"
>
> [JP Morgan Fourth Quarter 2014]

Note that the debate about FVA is connected to the recognition (or not) of debt value adjustment (DVA) which represents the value from a firm's own default. DVA can be seen to be associated to FVA as a more economically meaningful funding benefit. This circumvents problems with DVA such as banks booking profits when their own credit quality declines (Wilson 2011). One important aspect in the FVA/DVA debate is wealth transfer effects between shareholders and creditors. Initial margin and MVA will likely lead to similar debates and discussion of the underlying wealth transfer mechanisms.

Despite similarities relating to funding costs, FVA and MVA are rather different in several aspects. FVA is generally a representation of costs arising from posting (or not receiving) variation margin. MVA is associated with initial margin, which corresponds to overcollateralisation and will therefore give rise to other effects. Notably creditors holding initial margin will effectively recover more in a default situation, even if they are pari passu[†] with other creditors. Hence, initial margin makes derivatives creditors structurally senior to other creditors. Since initial margin represents overcollateralisation, it also needs to be

[†] Of the same seniority in terms of claims in the event of default.

segregated to protect the poster from incurring greater counterparty risk. This segregation makes remuneration of posted initial margin difficult. These aspects mean that the effective cost of funding initial margin may differ from funding costs assumed for FVA.

In this article, we analyse the impact of initial margin. We discuss the mechanics of initial margin calculations and discuss some of the likely implications and potential problems associated with increased initial margin posting. We show that there is a wealth transfer mechanism from other creditors to derivatives creditors. Using a structural model, we illustrate the impact of this wealth transfer via components such as the loss given default and credit spreads of different creditors. We show that unsecured creditors should charge banks more for lending money due to the expectation that some of this money will be pledged to OTC derivatives counterparties in the form of initial margin. We also suggest that this may lead to bespoke funding structures for raising initial margin.

2. Background

2.1 Funding costs and wealth transfer effects

In recent years, banks have begun to charge FVA to clients for the funding costs inherent in (mainly uncollateralised) derivatives transactions. FVA has become synonymous with the more traditional (and less controversial) credit value adjustment (CVA). The work of Piterbarg (2010) and Burgard and Kjaer (2011) highlighted FVA from a theoretical viewpoint as the incorporation of the funding costs and benefits into the value of a derivatives transaction. The adoption of FVA has been reasonably swift and anecdotally it is seen in most market prices quoted to clients. Furthermore, FVA accounting adjustments have also been made in the financial statements of most major banks. Note that the question over FVA is interrelated with a similar question on the relevance of DVA. Arguments supporting FVA would tend to avoid DVA on the basis that it is already incorporated in the former as a funding benefit.

Despite the quite rapid incorporation of FVA in pricing, some authors have questioned its validity, notably Hull and White (2012, 2014) and Andersen et al. (2016). These frameworks tend to support notions such as DVA and then conform to price symmetry and the law of one price. One way[‡] to interpret these arguments is that FVA is merely an internal transfer of wealth within a firm and therefore should not need to be represented in that firm's financial statements. Even agreeing with this argument, there is still justification for a bank charging a client FVA. One example is that part of the funding cost should be charged in the form of a funding risk premium which is the component of the funding cost that is not related to the credit risk of the party concerned (Morini and Prampolini 2011). Hull and White (2014) support this by stating the FVA is "*justifiable only for the part of a company's credit spread that does not reflect default risk*". Andersen et al. (2016) also justify FVA from a pricing (but not accounting) perspective by showing that it arises from the maximisation of shareholder value. Since shareholders pay funding costs that represent a windfall to creditors in the event of default (Burgard and Kjaer 2011), an FVA charge in a transaction acts as a compensation to shareholders for this wealth transfer effect. Whether or not the presence of such charges in transaction prices justifies its use as an accounting adjustment remains a debate. Some would argue that the definition of fair value as an exit price under IFRS 13 does indeed support this notion. After all, a party's exit price is another party's entry price.

Not surprisingly, MVA represents similar problems to FVA in terms of its validity in pricing and as an accounting adjustment. As with FVA, Andersen et al. (2016) show that the overall

[‡] We note that there are other arguments, such as arbitrage considerations made by Hull and White (2014).

effect of initial margin posting has no net impact on the total value of a firm and these authors therefore argue that it should not be a component of financial statements. However, analogously to FVA, charging MVA to a client can be justified from a shareholder-centric point of view that provides compensation for the wealth transfer. Hence, it seems likely that MVA will receive much attention in the coming years similar to recent interest in FVA.

However, MVA differs from FVA for the following reasons:

- FVA charges are normally imposed directly on a client and relate to the actual transaction in question and the fact that the client is not (perfectly) collateralising that transaction. As such, FVA charges can often be determined without reference to other transactions. By contrast, initial margin requirements are generally only relevant between financial institutions who cannot charge each other. Traditional end-user clients are exempt from centrally clearing trades or posting initial margin bilaterally.[§] Hence, an MVA charge may not relate to an actual client transaction but rather to the hedge(s) that arise in order to neutralise the market risk of the transaction. For example, an end-user client may be charged CVA and FVA on the basis that they are (partially) uncollateralised but also an MVA due to the fact that the hedge of the transaction attracts initial margin requirements (either as a result of central clearing or the bilateral margin rules).
- It has been argued (e.g. see Albanese 2015) that FVA can be reduced since regulatory capital can be used to fund variation margin payments. This argument would not apply to MVA since initial margin is pledged by title transfer and is therefore not interchangeable with capital.
- Whereas FVA involves a wealth transfer between shareholders and creditors, the analogous MVA benefit applies only to creditors that benefit from initial margin, namely derivatives creditors. Hence, MVA should be seen as a wealth transfer to derivatives creditors from other creditors. This may in turn create agency problems where unsecured lenders may struggle to gauge the remuneration they require unless they understand to what extent they are subordinated to derivatives creditors in the event of a default.

This paper will use a structural model to illustrate the latter point above showing the impact of initial margin posting on derivatives creditors and other creditors (termed generically as bondholders). This is important since regulatory changes such as the clearing mandate and bilateral margin requirements aim to reduce systemic risk via initial margin posting. However, whilst initial margin reduces the counterparty risk losses on derivatives trades, it increases losses of other claimants (Pirrong 2013). Whilst initial margin might therefore reduce systemic risk in derivatives markets (due to smaller default losses), it may increase it in these other markets where greater losses may have to be absorbed.

2.2 Liquidity impact of initial margin

Initial margin aims to create a "defaulter pays" environment where a defaulted counterparty pays for claims a priori via pledging initial margin which is held in a segregated account (to prevent it being used for other purposes thereby increasing counterparty risk). Initial margin is a concept that developed on derivatives exchanges to cover the close-out costs of relatively short-dated and liquid transactions. When applied to OTC derivatives, the underlying maturity transformation that occurs is more stark as illustrated in Figure 1. This shows a five-year maturity transaction in situations without and with initial margin. In the former case, aspects

[§] Note that in such a situation their counterparty would also be exempt from posting initial margin (i.e. bilateral margin posting is not one-way).

such as credit exposure, potential future exposure (PFE), CVA and regulatory capital tend to be assessed with the entire lifetime of the transaction in mind. The objective of initial margin is to mitigate all of these aspects to a high confidence level (99% or more). The time horizon for initial margin calculations is short: typically 10-days under the bilateral margin rules and 5-days for central clearing.[**] Such short horizons are partly justified by the assumption that a surviving party (central or bilateral) should be able to close-out and replace an OTC derivatives portfolio within such a time horizon. Furthermore, since initial margin is dynamically recalculated on a daily basis, it can potentially increase where necessary in, for example, more volatile market conditions. However, this does raise a question over the potential liquidity impact of such increasing initial margin requirements.

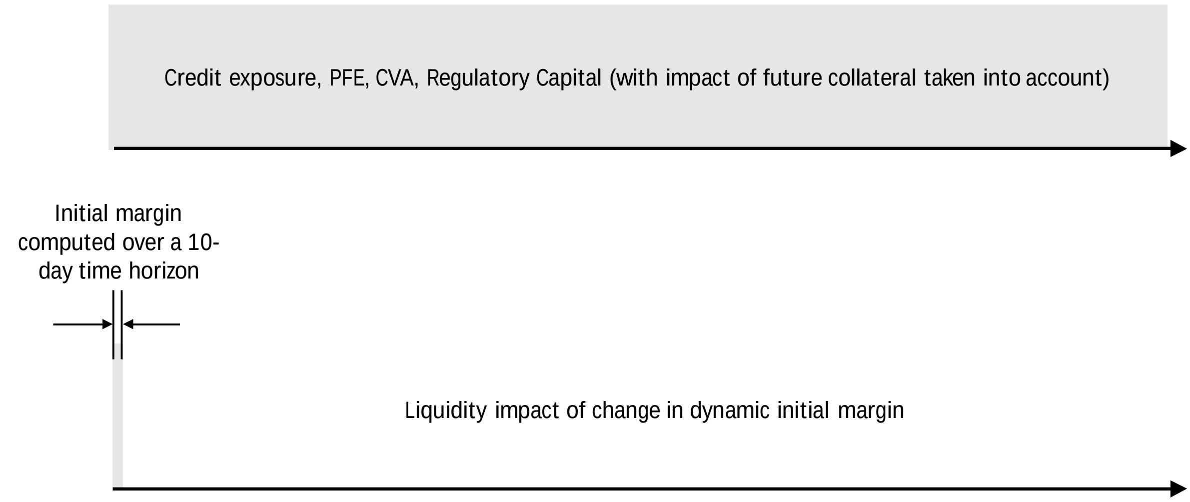

Figure 1. *Impact of the use of initial margin in OTC derivatives transactions. The top picture depicts a five-year transaction without initial margin posting, and the bottom shows the same transaction with initial margin assuming the time horizon for the calculation is 10-days.*

Clearly, the methodology for determining initial margin is important. On the one hand, initial margin should be a dynamic quantity and adjust for changes in portfolio composition (for example, transactions rolling off) and changing (e.g. more volatile) market conditions. Not surprisingly, value-at-risk (VAR) approaches have become relatively common to tackle the underlying complexity and multidimensionality of OTC derivatives portfolios in this respect. Such approaches employ high confidence levels and tend to be calibrated using historical data and therefore are in danger of being pro-cyclical by prescribing lower initial margin during non-volatile market conditions and vice versa. Regulation and market practice seeks to reduce such pro-cyclicality by requiring the use of, for example, stressed periods of data and longer time horizons. Whilst such undesirable features can be mitigated against, they cannot be avoided completely: for example, the volatility in the credit default swaps market in the aftermath of the Lehman Brothers bankruptcy was five times greater than in the preceding period (Pykhtin and Rosen 2012). Any unprecedented event in a given asset class will almost by construction lead to a sudden increase in initial margin requirements. This was illustrated recently as a result of significant changes in Sterling swap rates resulting from the result of the British referendum on European Union membership on 30th June 2016. Such moves were unprecedented in the sense that they caused almost immediate increased initial margin requirements for interest rate swaps in this currency.

[**] These values tend to be smaller for shorter-dated and more liquid exchange-traded derivatives with 1- or 2-day being common.

2.3 Risk mitigation effect of initial margin

Since initial margin is taken at a high confidence level then it should be expected to reduce credit exposure and associated CVA and regulatory capital charges[††] so as to become negligible. In order to understand this, assume that the change in value of a derivative portfolio during the time taken to receive collateral – the so-called margin period of risk (MPR) - is given by $\Delta V = \mu' + \sigma' Z$ with μ' and $\sigma' = \sigma\sqrt{\tau_{MPR}}$ being drift and volatility parameters respectively, τ_{MPR} representing the length of the MPR and Z being a standard normal variable. The expected exposure (EE) under such normal distribution assumptions is (for example, see Gregory 2015):

$$EE = \mu'\Phi\left(\frac{\mu'}{\sigma'}\right) + \sigma'\varphi\left(\frac{\mu'}{\sigma'}\right) \tag{1}$$

where $\varphi(.)$ represents a normal distribution function and $\Phi(.)$ represents the cumulative normal distribution function.

Assuming that initial margin is calculated over a time horizon τ_{IM} and with normal distribution assumptions then the impact of (instantaneously received) initial margin is equivalent to shifting the mean of the distribution to be $\mu' = -\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}$ where α is the confidence level used in the initial margin calculation. From equation (1), this leads to an EE of:

$$EE_{IM} = -\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}\Phi\left(\frac{-\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}}{\sqrt{\tau_{MPR}}}\right) + \sqrt{\tau_{MPR}}\varphi\left(\frac{-\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}}{\sqrt{\tau_{MPR}}}\right)$$

This can be simplified to give:

$$EE_{IM} = \sqrt{\tau_{MPR}}\varphi\left(\sqrt{\lambda}K\right) - K\sqrt{\tau_{IM}}\Phi\left(-\sqrt{\lambda}K\right) \tag{2}$$

where $K = \Phi^{-1}(\alpha)$ and $\lambda = \tau_{IM}/\tau_{MPR}$ is the ratio of the time horizon used (τ_{IM}) for the IM calculation divided by the MPR for the exposure quantification (τ_{MPR}). The value of λ may be unity since bilateral margin rules (BCBS-IOSCO 2015) align with Basel III regulatory capital rules (BCBS 2013) in requiring $\tau_{IM} = \tau_{MPR} = 10$ (business days) in standard cases. However, other values may be possible: for example, Basel III capital requirements require the MPR to be doubled (or more) in some cases, leading to $\lambda = 0.5$.

Without initial margin, we would simply have $\mu' = 0$ and therefore obtain:

$$EE_{no\ IM} = \sqrt{\tau_{MPR}}\varphi(0) = \sqrt{\frac{\tau_{MPR}}{2\pi}} \tag{3}$$

Taking the ratio of the EE with and without initial margin will give an approximation for the relative reduction (R_α):

$$R_\alpha = \frac{EE_{no\ IM}}{EE_{IM}} = \left[\sqrt{2\pi}\left(\varphi\left(\sqrt{\lambda}K\right) - K\sqrt{\lambda}\Phi\left(-\sqrt{\lambda}K\right)\right)\right]^{-1} \tag{4}$$

Table 1 shows the reduction in EE calculated at various confidence levels and values of λ. For example, initial margin at the 99% confidence level will reduce the exposure by over two orders of magnitude (117.7 times) if the time horizons are the same.

[††] Noting that this will depend on the precise calculation methodology used. Regulatory capital rules can be relatively conservative in their treatment of aspects such as initial margin with the SA-CCR approach (BCBS 2014), for example, imposing a floor of 5% on the beneficial risk reduction of initial margin.

Table 1. *EE for a normal distribution with IM at different confidence levels. The ratio is defined as the EE with no IM divided by the ratio with IM.*

	$\lambda = 1$	$\lambda = 0.5$
90%	8.4	4.0
95%	19.1	6.6
99%	117.7	19.1
99.5%	252.4	29.5

The above simple example suggests that initial margin should make residual credit exposure practically zero and give derivatives creditors absolute seniority over other creditors. However, this will not actually be the case in reality. In practical terms, initial margin may be insufficient to meet the claims of the derivatives creditors for the following reasons (note that centrally cleared transactions are impacted only by the first point but that all points are relevant for bilateral transactions with initial margin):

- *Confidence level.* Initial margin calculations are generally dynamic and model driven with a confidence level of 99% or more. Furthermore, the parametrisations of such models are generally unconditional (although they may use concepts such as stressed periods). Hence, it is possible that the (conditional) loss in the event of a significant default may potentially be underestimated by the initial margin amount as argued above. Note that initial margin posting entities are likely to be large and potentially systemically important financial counterparties and may have a significant impact on market volatility and liquidity (for example, see Pykhtin and Sokol 2012).
- *Threshold.* The bilateral margin rules allow a threshold of up to €50m for each consolidated entity against which initial margin does not need to be posted. Hence, against an initial margin requirement of €70m, only €20m actually needs to be pledged.
- *Coverage.* The bilateral margin rules exempt certain transactions such as some FX (including the FX component of cross-currency swaps). Such transactions would likely fall under the same legal terms as those for which initial margin is required but would not themselves attract any initial margin requirement.
- *Phase-in.* The bilateral margin rules are phased-in with the largest counterparties affected first. Furthermore, even when a firm falls under the rules then initial margin requirements are mandatory only for new, and not legacy, transactions. Clearly this effect will decrease over time although, given many OTC derivatives transactions are long-dated, then it will take many years for this point to become irrelevant.

Another point to note is the conditionality implied by initial margin which is not explicitly recognised in methodologies. Traditional application of VAR for the market risk of the trading book of a bank is unconditional in the sense that it is designed to assess losses that may occur on any given day. Initial margin, on the other hand, should be conditional in that it is required only in a default situation. If such a default involves a large OTC derivatives counterparty, then clearly the underlying market conditions would be expected to be anything but usual. To take an example, LCH.Clearnet's SwapClear clearing offering uses a historical VAR methodology called PAIRS[‡‡] to calculate initial margin. PAIRS subjects a given portfolio to approximately the last 10-years (2,500 days) of data and then defines the initial margin as the average of the worst six losses that would have occurred. On the face of it, unconditionally, this represents a

[‡‡] Portfolio Approach to Interest Rate Scenarios. See http://www.lchclearnet.com/risk-collateral-management/margin-methodology/pairs.

confidence level of at least 99.76%.[§§] However, given the initial margin is needed only in the event of a default then it might be argued that the confidence level on a conditional basis is much lower. Indeed, it is likely that at least some of the six worse scenarios in this example may involve daily move from around the aftermath of the last major derivatives counterparty default (Lehman Brothers). From this point of view, one would expect the initial margin to be closer to the median that the 99.76% quantile.

Initial margin methodologies could potentially represent the worst of both worlds. On the one hand, the use of (unconditional) tail measures with high confidence levels to drive requirements increases the likelihood that such quantities may be quite variable over time. On the other hand, by not being explicitly conditioned to the potential consequence of a large OTC derivatives counterparty default, the initial margin amounts may not be as protective as seem to be implied.

Furthermore, although not discussed here, the incorporation of dynamic initial margin into calculations of quantities such as CVA is extremely challenging since the initial margin methodology is generally simulation or sensitivity based. Green and Kenyon (2015) discuss this in more detail.

2.4 Initial margin and partial seniority

As noted above, initial margin posting will effectively make derivatives creditors more senior to other creditors since they may use the initial margin to offset losses in a default scenario. However, they may not be completely senior in the traditional sense (i.e. paid in full before other creditors will receive anything) as the coverage of initial margin cannot be 100% and may be significantly smaller as argued in the last section.

Let us illustrate the partial seniority with a simple example. Suppose a firm defaults and derivatives creditors have a claim of 15 with other creditors being owed 45. The residual firm value is 20 but the derivatives creditors have recourse to further initial margin of 10. In this case, derivatives creditors will use the full initial margin to cover their losses and claim the residual 5 alongside the other creditors. The recovery rate will then be 40% with derivatives creditors paid an additional 2 and other creditors receiving 18. This assumes that, notwithstanding initial margin, the creditors have the same seniority. As shown in Table 2, derivatives creditors therefore receive a much higher recovery rate at the expense of other creditors who are paid proportionally less.

[§§] This calculation represents an expected shortfall measure at the $1 - \frac{6}{2500} = 99.76\%$ confidence level. By construction, expected shortfall is more conservative than VAR which would simply take the 6th (worst) highest loss. Note that PAIRS incorporates volatility scaling which will lead to increased margin requirements in more volatility market conditions.

Table 2. Illustration of the impact of initial margin posting on the claims of derivatives and other creditors.

	Claim	Initial Margin	Recovery	Effective recovery rate
Derivatives creditors	15	10	2	80%
Other creditors	45	-	18	40%
Total	60	10	20	50%

The above example illustrates the fact that initial margin posting gives derivatives creditors partial seniority over other creditors. Obviously, if the initial margin is sufficient to cover the claims of the derivatives creditors then the seniority will be absolute.

3. Structural model

3.1 Assumptions

The classic Merton (1974) approach models the value of a firm as a stochastic process and then values equity and debt as options on the firm value. It has been extended to cases where there are different payoffs such as safety covenants and debt subordination (Black and Cox 1976), convertible bonds (for example, see Brennan and Schwartz 1977, 1980) and convertible debt (for example, see Glasserman and Nouri 2012). The analysis below shares some common components with previous work but is specific to the case of initial margin posting.

We consider a firm with total value given by a combination of a firm value $\widetilde{V}_t$ and an initial margin, M_t which is posted to certain derivatives creditors (bilateral counterparties and central counterparties). The firm value is assumed to follow a traditional Merton-style geometric Brownian motion and the initial margin is assumed to be segregated and earn the risk-free rate of interest, i.e.:

$$V_t = \widetilde{V}_t + M_t \tag{5a}$$

$$d\widetilde{V}_t = \mu\widetilde{V}_t dt + \sigma\widetilde{V}_t dW_t \tag{5b}$$

$$dM_t = rM_t dt \tag{5c}$$

where μ, σ and r are the usual drift, volatility and risk-free interest rate respectively. We assume a single class of derivatives creditors have a net claim of F^D and that there are a single class of bondholders with a claim of F^B. In the event of default at the assumed maturity date T, it is assumed that the derivatives creditors can freely use the initial margin to cover their claims. If there is excess initial margin, then this will be returned to the bondholders. In the event that there is insufficient initial margin to cover the derivatives creditor's claims then they and the bondholders will have an equal priority over the remaining firm value.

3.2 Payoffs

Define α^D and α^B respectively as the percentage claim of the derivatives creditors and bondholders in the event of default as:

$$\alpha^D = \frac{(F^D - M_T)_+}{F^D + F^B - M_t} = 1 - \alpha^B, \qquad (6)$$

where $(x)_+ = \max(x, 0)$. Note that if the initial margin is sufficient to cover the claims of the derivatives creditors then any residual amount will pass to the bondholders as it would be returned to the estate of the bankrupt firm.

Default is assumed to occur at the maturity date T if the firm is unable to meet all the claims of creditors: i.e. where $V_T = \tilde{V}_T + M_T < F^D + F^B$. However, note that in this situation there are two possible scenarios:

i) $M_T \geq F^D$. In this case, the derivatives creditors have sufficient initial margin to cover their claims and will therefore suffer no loss. The bondholders will recover the firm value and any residual initial margin, i.e. they will receive the value of $\tilde{V}_T + M_T - F^D$ (which will be less than the value of their claim of F^B in this case).

ii) $M_T < F^D$. In this case, the derivatives creditors are not paid in full and experience losses together with bondholders although they will experience different recoveries in the case of $M_T > 0$. Note that the derivatives creditors are not completely senior with respect to the bondholders in this case.

Obviously, the shareholder will receive any residual value. The payoffs are given in Table 3.

Table 3. *Payoffs to derivatives creditors, bondholders and shareholders in the presence of initial margin.*

	$\tilde{V}_T + M_T < F^D + F^B$		$\tilde{V}_T + M_T \geq F^D + F^B$
	$M_T < F^D$	$M_T \geq F^D$	
Derivatives	$M_T + \alpha^D \tilde{V}_T$	F^D	F^D
Bondholders	$\alpha^B \tilde{V}_T$	$\tilde{V}_T + M_T - F^D$	F^B
Shareholders	0		$\tilde{V}_T + M_T - F^D - F^B$
Total	$\tilde{V}_T + M_T$		$\tilde{V}_T + M_T$

3.3 Valuation

As in the tradition Merton model, shareholders have a call option payoff on the firm value:

$$E_T = [\tilde{V}_T + M_T - F^D - F^B]_+ \qquad (7)$$

The underlying value is given by:

$$E_t = \tilde{V}_0 \Phi(d_1) - (F^D + F^B - M_T)e^{-r(T-t)}\Phi(d_2) \qquad (8)$$

$$d_1 = \frac{\ln\left(\frac{\tilde{V}_t}{F^B + F^C - M_T}\right) + \left(r + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}} = d_2 - \sigma\sqrt{T - t}$$

Where $(T-t)$ represents the time to maturity and $M_T = M_t e^{r(T-t)}$. The derivatives creditors have a payoff defined by:

$$D_T = \min\left(F^D, M_T + \alpha^D \tilde{V}_T\right) = F^D - [F^D - M_T - \alpha^D \tilde{V}_T]_+ \tag{9}$$

The derivatives creditors are therefore short a put option which represents the possibility that the initial margin and their pro-rata share of the residual firm value may be insufficient to meet their claim. The value of the derivatives creditor's claim can therefore be written as:

$$D_t = F^D e^{-r(T-t)} - \left[(F^D - M_T)_+ e^{-r(T-t)} \Phi(-d_2) - \alpha^D \tilde{V}_0 \Phi(-d_1)\right] \tag{10}$$

In the case where $F^D \geq M_T$ then this becomes:

$$= F_D e^{-r(T-t)} - \left[(F^D - M_T) e^{-r(T-t)} \Phi(-d_2) - \alpha^D \tilde{V}_0 \Phi(-d_1)\right]$$

$$= F_D e^{-r(T-t)} - \alpha^D \left[(F^D + F^B - M_t) e^{-r(T-t)} \Phi(-d_2) - \tilde{V}_0 \Phi(-d_1)\right] \tag{11}$$

This is a classic result where the bondholders are short a put option on a loss fraction of α^D. This loss fraction is smaller than their pro-rata claim, due to their seniority created by the initial margin.

Bondholders will receive a pro-rata claim on the firm value and may receive more than this if the derivatives creditors do not have a shortfall in their claim (i.e. the initial margin is high enough). The payoff for the bondholders will be:

$$B_T = \tilde{V}_T + M_T - [V_T + M_T - F^D - F^B]_+ - F^D + [F^D - M_T - \alpha^D \tilde{V}_T]_+$$

$$= F^B - \max\left(F^D + F^B - \tilde{V}_T - M_T, 0\right) + \max(F_D - M_T - \alpha^D \tilde{V}_T, 0) \tag{12}$$

The first term represents the usual short put option since the bondholders will only be paid in full if the total firm value is greater than the total debt claims. The second term is a long put option which has a strike at the point where derivatives creditors are paid in full. The total value can therefore be written as:

$$B_t = F^B e^{-r(T-t)} - (F^B + F^D - M_T) e^{-r(T-t)} \Phi(-d_2) + \tilde{V}_0 \Phi(-d_1) \\ + (F^D - M_T)_+ e^{-r(T-t)} \Phi(-d_2) - \alpha^D \tilde{V}_0 \Phi(-d_1^*)$$

$$= F^B e^{-r(T-t)} - [(F^B + F^D - M_T) - (F^D - M_T)_+] e^{-r(T-t)} \Phi(-d_2) \\ + \alpha^B \tilde{V}_0 \Phi(-d_1) \tag{13}$$

Note that in the case where $F^D \geq M_T$ then this becomes:

$$F^B e^{-r(T-t)} - \alpha^B (F^D + F^B - M_T) e^{-r(T-t)} \Phi(-d_2) + \alpha_B \tilde{V}_0 \Phi(-d_1) \tag{14}$$

This is a classic result where the bondholders are short a put option but on a higher (than pro-rata) loss fraction of α_B due to the structural seniority of the derivatives creditors.

A final important result which will be used for comparison purposes is where the derivatives creditors and bondholders are assumed to have the same seniority and the initial margin is shared pro-rata. In this case the valuation would give:

$$B_t^* = F^B e^{-r(T-t)} - \alpha^{B^*} (F^D + F^B - M_T) e^{-r(T-t)} \Phi(-d_2) + \alpha^{B^*} \tilde{V}_0 \Phi(-d_1) \tag{15}$$

$$\alpha_B^* = \frac{F^B}{F^D + F^B}$$

3.4 Loss given default

The default probability (applicable to both bondholders and derivatives creditors as mentioned above) can be written as:

$$Q_T = \Pr\left(\tilde{V}_T + M_T < F^B + F^C\right) = \Phi(-d_2) \tag{16}$$

Due to the structural seniority of the derivatives creditors, they may be paid in full when bondholders are not. This will be represented in terms of the loss given default being zero. Re-writing the value of the derivatives creditors in equation (10) as:

$$D_t = F^D e^{-r(T-t)} \left[1 - \Phi(-d_2)\left[\frac{(F^D - M_T)_+}{F^D} - \frac{\alpha^D \tilde{V}_0 \Phi(-d_1)}{F^D e^{-r(T-t)} \Phi(-d_2)}\right]\right]$$

We can therefore identify the percentage expected loss given default (LGD) as:

$$LGD_t^D = \frac{1}{F^D}\left[(F^D - M_T)_+ - \frac{\alpha^D \tilde{V}_T \Phi(-d_1)}{e^{-r(T-t)} \Phi(-d_2)}\right] \tag{17}$$

Doing the same thing for the bondholders from equation (13) yields:

$$B_t = F_B e^{-r(T-t)} \left[1 - \Phi(-d_2)\left[\frac{(F_B + F_D - M_T) - (F_D - M_T)_+}{F^B} + \frac{\alpha^D \tilde{V}_T \Phi(-d_1)}{F^B e^{-r(T-t)} \Phi(-d_2)}\right]\right]$$

The expected loss given default is therefore:

$$LGD_t^B = \frac{1}{F^B}\left[(F^B + F^D - M_T) - (F^D - M_T)_+ - \frac{\alpha^B \tilde{V}_0 \Phi(-d_1)}{e^{-r(T-t)} \Phi(-d_2)}\right] \tag{18}$$

In the case where the bondholders are of the same seniority (used for comparison purposes) we can obtain (from equation 15):

$$X^{B^*} = F^B e^{-r(T-t)} \left[1 - \Phi(-d_2)\left[\frac{\alpha^{B^*}(F^D + F^B - M_T)}{F^B} - \frac{\alpha^{B^*} \tilde{V}_T \Phi(-d_1)}{F^B e^{-r(T-t)} \Phi(-d_2)}\right]\right]$$

The expected loss given default is therefore:

$$LGD_t^{B^*} = \frac{1}{F^B}\left[\alpha^{B^*}(F^D + F^B - M_T) - \frac{\alpha^{B^*} \tilde{V}_T \Phi(-d_1)}{e^{-r(T-t)} \Phi(-d_2)}\right] \tag{19}$$

3.5 Credit spreads

In the Merton approach, implied credit spreads can be derived by solving for the spread in the following formula:

$$X_t = F^X \exp\left[-\left(r + s_{t,T}\right)(T - t)\right]$$

$$s_{t,T} = -\frac{ln(X_t / F^X)}{(T - t)} - r$$

where $s_{t,T}$ is the credit spread at time t for a maturity date T. Doing this for the derivatives creditors gives:

$$s_{t,T}^{D} = \frac{-ln\left[e^{-r(T-t)} - \frac{1}{F^D}\left[(F^D - M_T)_+ e^{-r(T-t)}\Phi(-d_2) - \alpha^D \tilde{V}_0 \Phi(-d_1)\right]\right]}{T-t} - r$$

$$= -\frac{1}{T-t} ln\left[1 - \frac{(F^D - M_T)_+}{F^D}\Phi(-d_2) + \frac{\alpha^D \tilde{V}_0 \Phi(-d_1)}{F^D e^{-r(T-t)}}\right] \qquad (20)$$

For the bondholders a similar analysis gives:

$$s_{t,T}^{B} = -\frac{1}{T-t} ln\left[e^{-r(T-t)} - \frac{[(F^B + F^D - M_T) - (F^D - M_T)_+]}{F^B} e^{-r(T-t)}\Phi(-d_2) + \frac{\alpha^B \tilde{V}_0 \Phi(-d_1)}{F^B}\right] - r$$

$$-\frac{1}{T-t} ln\left[1 - \frac{[(F^B + F^D - M_T) - (F^D - M_T)_+]}{F^B}\Phi(-d_2) + \frac{\alpha^B \tilde{V}_0 \Phi(-d_1)}{F^B e^{-r(T-t)}}\right] \qquad (21)$$

Finally, for the comparison case where the bondholders have an equal claim on the initial margin:

$$s_{t,T}^{B^*} = -\frac{1}{T-t} ln\left[1 - \frac{(F_D + F_B - M_T)}{F^D + F^B}\Phi(-d_2) + \frac{\tilde{V}_t \Phi(-d_1)}{(F_D + F_B)e^{-r(T-t)}}\right] \qquad (22)$$

3.6 Results

We will use some examples to illustrate the structural seniority of derivative creditors via lower expected loss given default and implied credit spreads. We first show the impact of initial margin via the formulas developed above. Then in Figure 2 we show the LGD for derivatives creditors and bondholders as given by equations (17) and (18) and compare to the reference case where creditors have equal claim on the initial margin (equation 14). This shows that the impact of initial margin is to improve (lower) the LGD of the derivatives creditors at the expense of the bondholders who suffer a higher LGD compared to the reference case.

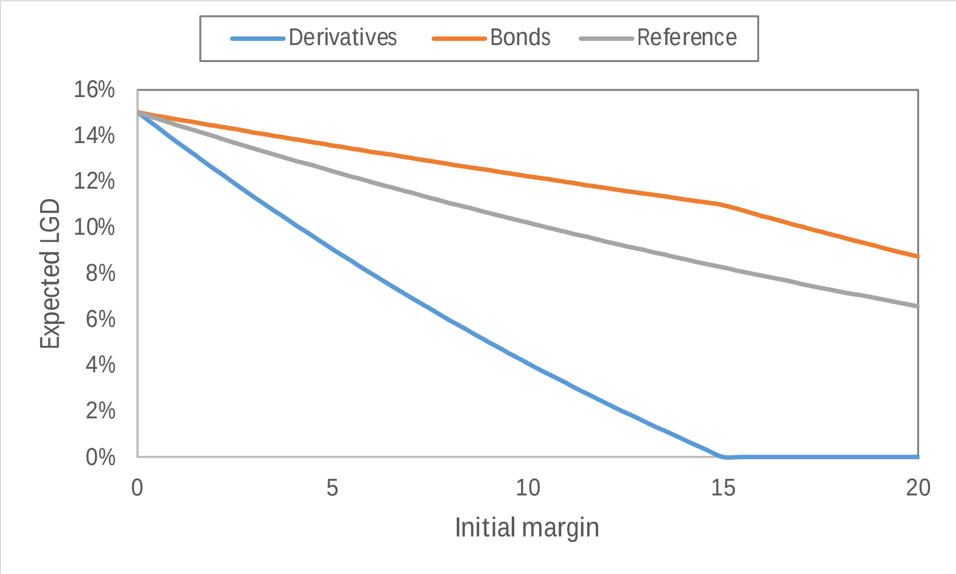

Figure 2. *Expected loss given default for derivatives creditors and bondholders as a function of the initial margin posted against derivatives claims and compared to the reference case where the initial margin is shared pro-rata. Parameters used:* $\widetilde{\boldsymbol{V}}_{\mathbf{0}} = \mathbf{75}$, $\boldsymbol{F}^{\boldsymbol{D}} = \mathbf{15}$, $\boldsymbol{F}^{\boldsymbol{B}} = \mathbf{45}$, $\boldsymbol{\sigma} = \mathbf{30}\%$, $(\boldsymbol{T} - \boldsymbol{t}) = \mathbf{1}$, $\boldsymbol{r} = \mathbf{5}\%$.

Initial margin can be seen to give derivative creditors structural seniority with respect to other creditors. This implies that lenders should demand a greater return for lending money to firms that may be subject to initial margin posting either via the central clearing mandate or the bilateral requirements. Another way to look at this is via the implied credit spreads as a function of maturity for derivatives creditors and bondholders as given by equations (20) and (21) and compared to the reference case (equation 22). This is shown in Figure 3 for different initial margin amounts. As the initial margin increases, the credit spread of the derivatives creditors tightens and that of the bondholders widens with respect to the reference case. When the initial margin is greater than the claims of derivatives creditors then their implied credit spread is zero. Creditors should demand an additional return for credit risk due to the fact that they are made more junior via the process of initial margin posting. However, in order to know how much return to expect then creditors would need to understand the amount and mechanism by which initial margin will be pledged over the term of their lending. Since incoming regulation mandates increasing initial margin posting then this is difficult to assess since they face ever greater losses in the event of default due to being made junior by the posting of initial margin.

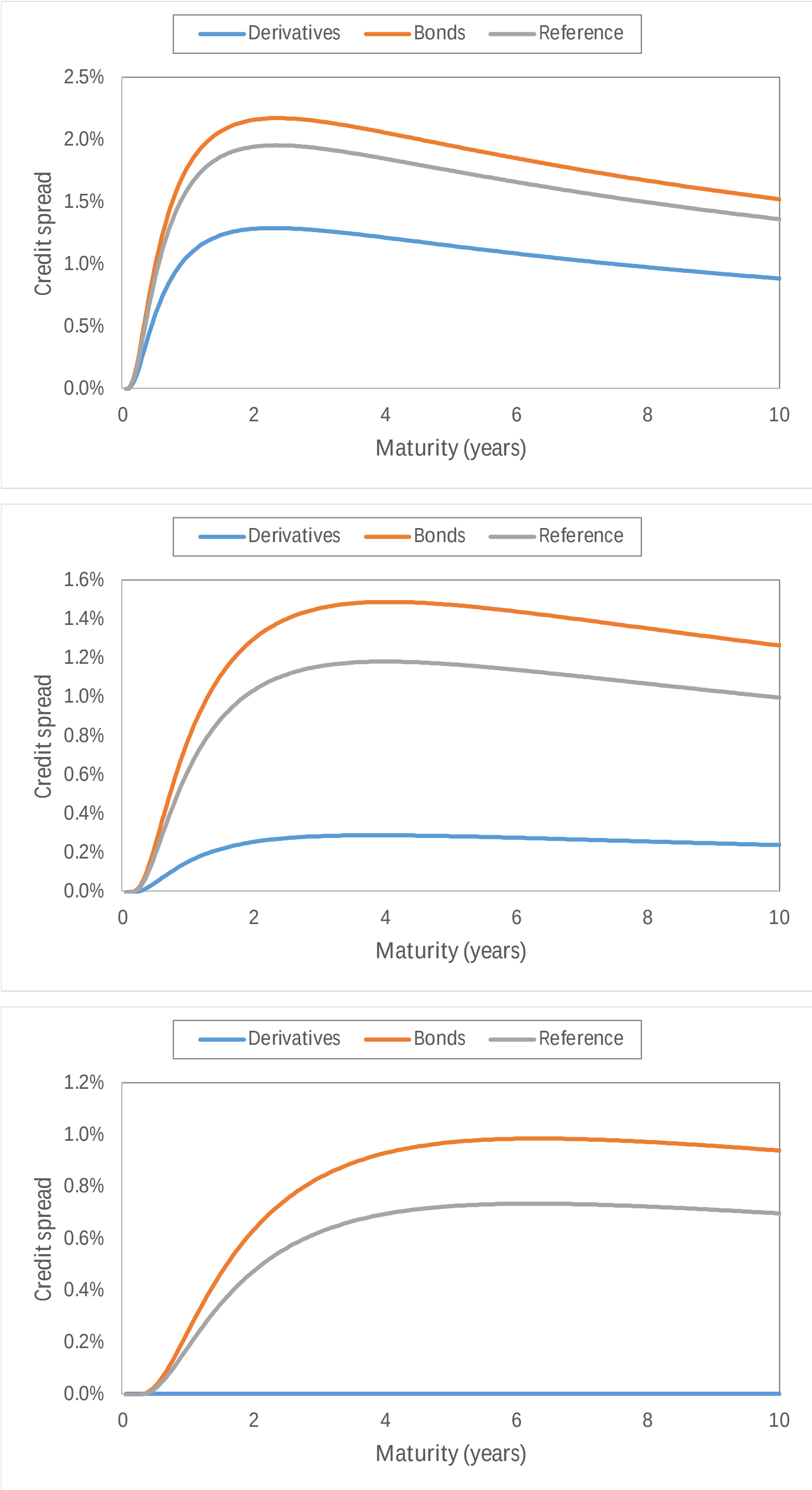

Figure 3. *Implied credit spreads for derivatives creditors and bondholders as a function of the initial margin posted against derivatives claims and compared to the reference case where the initial margin is shared pro-rata. Parameters used:* $\tilde{V}_0 = 75$, $F^D = 15$, $F^B = 45$, $\sigma = 30\%$, , $r = 5\%$. $M_0 = 6$ *(top), 12 (middle) and 18 (bottom).*

Finally, in Figure 4, we define the "juniorisation spread" as the difference between the credit spread of bondholders (equation 16) minus the reference case (equation 17). As the initial margin pledged increases then so does the additional spread that should be required by bondholders as compensation for credit risk. This is the case until the point at which the derivatives creditors will be paid in full (an amount of 15 in this example) whereupon the juniorisation spread declines again due to the fact that surplus initial margin is returned to the other creditors.

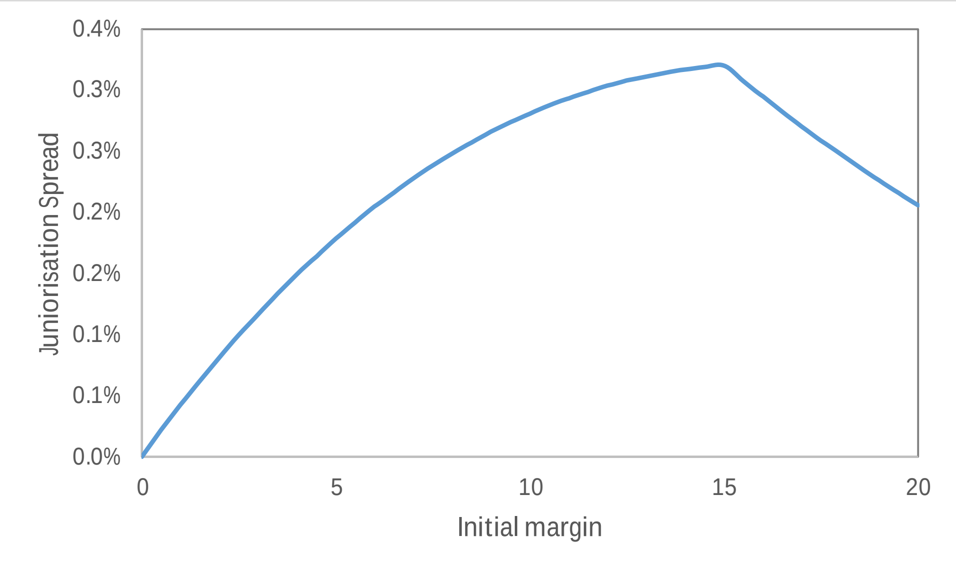

Figure 4. *Juniorisation spread defined as the difference between the credit spread of bondholders compared to the reference case as a function of initial margin. Parameters used:* $\tilde{V}_0$ = 75, F^D = 15, F^B = 45, σ = 30%, , r = 5%, $(T - t)$ = 5.

4. Conclusion

In this article, we have discussed the impact of initial margin posting by firms against OTC derivatives transactions as required by regulation such as the clearing mandate and bilateral margin rules. We have described the nature of initial margin, which is calculated at a high confidence level but over a very short time-horizon (compared to the maturity of many OTC derivatives transactions). We argue that the confidence levels used in initial margin methodologies may significantly underestimate the probability that an initial margin amount may be insufficient to cover the underlying losses in a default scenario. For this and other reasons, initial margin will give derivatives creditors only partial seniority over other creditors such as bondholders.

Posting segregated initial margin creates a wealth transfer between derivatives creditors and other creditors since the former receive a higher recovery in the event of default at the expense of the latter. We have described a structural model to illustrate the wealth transfer effect which can be used to show the relatively high loss given default experienced by other creditors as a result of initial margin posting. This model also shows the divergence of credit spreads for derivatives and other creditors: the latter tends to zero as the initial margin increased while the former experiences an increase. This implies, for example, that bondholders should charge more to banks in order to lend them money in the expectation that some of the funds might be used in order to finance initial margin posting. This, in turn, may lead to bespoke funding arrangements for initial margin as suggested by Albanese (2015), for example.

Given the increasing use of initial margin to support the derivatives trading activities of banks and major financial institutions, it will be important to understand the full impact in terms of aspects such as wealth and risk transfer effects. Initial margin posting is required by regulation because it supports the "defaulter pays" approach and should therefore reduce systemic risk. However, as shown in the result in this paper, what initial margin really does is to make derivatives creditors more senior at the expense of other creditors. Any systemic risk reduction will therefore be confined to derivatives markets only and there should be a commensurate increase in risks and costs in other markets. For example, it is interesting to study to what extent borrowing markets such as wholesale funding and repo markets might become risky due to the requirements on posting initial margin. This is particularly relevant since initial margin amounts are generally dynamic and will continuously change and may potentially increase in volatile or crisis periods. The resulting liquidity implications or effects are hard to predict and may create further destabilisation. Whether central clearing and bilateral margin posting actually reduce systemic risk and make financial markets safer overall remains an open question.

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