APPENDIX 17A: Simple wrong-way risk formula

More details of the calculations below can be found in Rosen and Pykhtin (2010).

i) EE for a forward contract under the assumption of a normally distributed MtM value

In Appendices 8B, we derived a simple formula for the expected exposure (EE) for an underlying MtM value (V_t) of the form:

$$dV_t = \mu dt + \sigma dW_t,$$

where μ represents a drift and σ is a volatility of the exposure with dW_t representing a standard Brownian motion. The expected exposure for a given time horizon s is given by (see Appendix 7A):

$$EE_s = \mu s \Phi\left(\frac{\mu}{\sigma}\sqrt{s}\right) + \sigma s \varphi\left(\frac{\mu}{\sigma}\sqrt{s}\right).$$

ii) EE expression conditional on default

Now we derive a similar formula but conditioning on some actual default time. Under the above assumptions, the value of a contract at some time s in the future is given by:

$$V(s) = \mu s + \sigma\sqrt{s}Y,$$

where Y is a Gaussian random variable. Let us denote the time of default of the counterparty by τ and the default probability of the counterparty up to time s as $F(s)$ which, as in Appendix 12A, is defined via a constant hazard rate h or intensity of default:

$$F(s) = 1 - \exp(-hs)$$

Like the exposure, default is driven by a Gaussian variable, Z:

$$\tau = F^{-1}(\Phi(Z))$$

Finally we link the Gaussian variables Y and Z via a correlation parameter ρ:

$$Y = \rho Z + \sqrt{1-\rho^2}\varepsilon,$$

with ε being a further (independent) Gaussian variable. We now need to calculate the expected exposure conditional upon default having occurred. This is:

$$EE(s|\tau = s) = E[\max(V(s), 0) | Z = \Phi^{-1}(F(\tau))]$$
$$= \int_{-\mu(t)/\sigma(t)}^{\infty} [\mu'(s) + \sigma'(s)]\varphi(x)dx$$

Where

$$\mu'(s) = \mu(s) - \rho\sigma\Phi^{-1}(F(\tau)) \qquad \sigma'(s) = \sqrt{1-\rho^2}\sigma$$

The expected exposure is then give by:

$$EE(s|\tau = s) = \mu'(s)\Phi\left(\frac{\mu'(s)}{\sigma'(s)}\right) + \sigma'(s)\varphi\left(\frac{\mu'(s)}{\sigma'(s)}\right).$$

This is illustrated in Spreadsheet 17.1.

APPENDIX 17B: Devaluation approach for FX wrong-way risk

We use the simple EE formula in Appendix 8A. Consider an FX contract where the FX volatility is 15%. The expected exposure of such a contract in 1 year in the case of no wrong way risk would be:

$$\$100m \times 15\% / \sqrt{2\pi} = \$5.98m \,.$$

Using the devaluation approach, suppose that we consider that if the counterparty were to default then the currency would be devalued to 80% of its previous value with a residual value (RV) of 20%. The expected exposure based on this wrong way risk assumption is:

$$\$100m \times \left[20\% \times \Phi(20\% / 15\%) + 15\% \times \varphi(20\% / 15\%)\right] = \$20.64m \,.$$

We see that in this example wrong way risk increases expected exposure at default by over a factor of three.

APPENDIX 17C: Credit default swaps with counterparty risk

Please see the following chapter (which is available on www.cvacentral.com) for more details.

Gregory J., 2011, "Counterparty risk in credit derivative contracts", The Oxford Handbook of Credit Derivatives, A. Lipton and A. Rennie (Eds), Oxford University Press.

APPENDIX 17D: Black-Scholes formula for counterparty risk (not listed in 3rd edition of the book)

The classic Black-Scholes formula for a European option can be extended to price a risky option (i.e. one that is extinguished when the counterparty defaults). We link the default time to a standard Gaussian random variable, Z :

$$\tau = F^{-1}\left(\Phi(Z)\right).$$

Now, with the standard Black-Scholes assumption for the evolution of the underlying asset (for example, a stock paying no dividends):

$$A(T) = A(t)\exp\left[(r - \sigma^2/2)T + \sigma\sqrt{T}X\right],$$

where $A(s)$ represents the asset price at time s , r is the risk-free interest rate, σ is the volatility, T is the option maturity and X is a standard Gaussian variable. The random term in the above expression is related to the default time via a correlation parameter ρ :

$$X = \rho Z + \sqrt{1-\rho^2}\,\varepsilon,$$

with ε being a further independent Gaussian variable. Let us consider the impact of positive correlation in this relationship. If the variable Z is very negative then τ will be small (default relatively soon) and the return on the asset is likely also to be negative. Hence, the asset is expected to be low when the counterparty defaults.

In this framework, it is possible to price risky options using expression similar to the Black-Scholes formulas. The pricing formulas for call (C) and put (P) options are given by:

$$C = e^{-rT}\left(F.A_1 - K.A_2\right) \qquad P = e^{-rT}\left(-F.A_{-1} + K.A_{-2}\right)$$

with the following definitions:

$$F = S\exp(rT), \qquad d_2 = \left[\ln(F/K) - \sigma^2/2)T\right]/\sigma\sqrt{T},$$

$$A_{\pm 1} = \int_{-\infty}^{\infty} \Phi\left(\pm\frac{\sqrt{\rho}u + \sigma\sqrt{T} + d_2}{\sqrt{1-\rho}}\right)\Phi\left(\frac{\sqrt{\rho}u + \rho\sigma\sqrt{T} - \Phi^{-1}\left(F(T)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du,$$

$$A_{\pm 2} = \int_{-\infty}^{\infty} \Phi\left(\pm\frac{\sqrt{\rho}u + d_2}{\sqrt{1-\rho}}\right)\Phi\left(\frac{\sqrt{\rho}u - \Phi^{-1}\left(F(T)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du.$$

In the case of zero default probability, $F(T) = 1$ and $\rho = 0$ we obtain $A_{\pm 1} = \sigma\sqrt{T} \pm d_2$ and $A_{\pm 2} = \pm d_2$ which correspond to the d_1 and d_2 terms in the standard Black-Scholes formula.

