APPENDIX 8A: LHP approximation and IRB formula

i) The LHP approximation

The large homogeneous pool (LHP) approximation of Vasicek (1997) is based on the assumption of a very large (technically infinitely large) portfolio. The loss distribution is defined via:

$$\Pr(L < \theta) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\theta) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right),$$

where $\Phi^{-1}(.)$ represents a cumulative normal distribution function, PD is the (constant) default probability and ρ the correlation parameter.

ii) The IRB formula details

The Basel II internal rating based (IRB) formula given in Equation (8.1) of the book is based on the above approximation together with the so-called granularity adjustment formula of Gordy (2004). This gives a worst case default probability which is defined by:

$$PD_{99.9\%} = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(99.9\%)}{\sqrt{1-\rho}}\right) - PD,$$

where the functions $\Phi(.)$ and $\Phi^{-1}(.)$ are the standard normal cumulative distribution function and its inverse.

The correlation parameter above, ρ , is linked to the default probability (PD) according to the following equation:

$$\rho = 0.12 \times \frac{1 - \exp(-50 \times PD)}{1 - \exp(-50)} + 0.24 \times \left(1 - \frac{1 - \exp(-50 \times PD)}{1 - \exp(-50)}\right)$$

This relationship is depicted in Figure 8.1A.

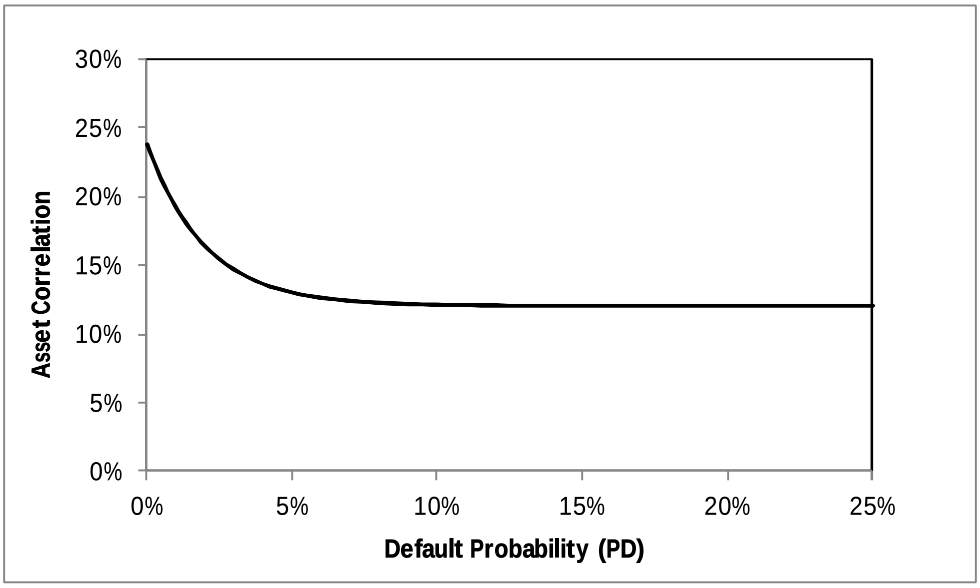

Figure 8.1A. *Correlation as a function of PD according to the IRB formula.*

In Equation (8.1) in the book, the factor $MA(PD, M)$ is the maturity adjustment that accounts for potential credit migration and is calculated from PD and M according to:

$$MA(PD, M) = \frac{1 + (M - 2.5) \times b(PD)}{1 - 1.5 \times b(PD)},$$

where $b(PD)$ is a function of PD defined as:

$$b(PD) = [0.11852 - 0.05478 \times \ln(PD)]^2.$$

Note that the maturity adjustment is capped at 5 and floored at 1. See Figure 8.2A for an example of this function.

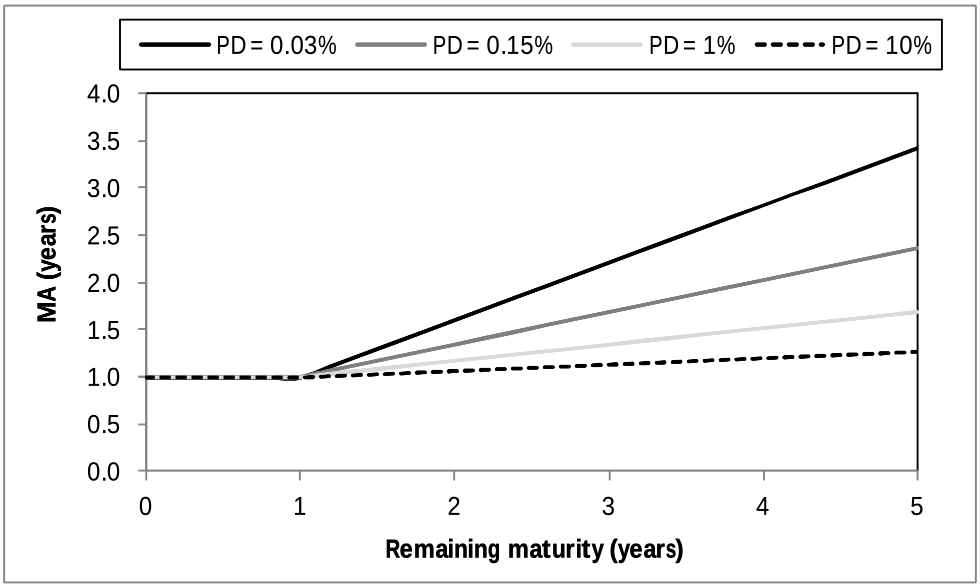

Figure 8.2A. *Maturity adjustment (MA) as a function of remaining maturity for several values of PD.*

The effective remaining maturity in the case of simple instruments such as loans with fixed unidirectional cashflows is defined as the weighed average maturity of the relevant transactions given by a simple duration formula without interest rate effects: $M = \sum_i CF_i t_i / \sum_i CF_i$, where CF_i is the magnitude of the cashflow at time t_i.

The cash flows of OTC derivatives are highly uncertain, and a more complex formula is required to calculate the effective maturity. This is therefore defined at the netting set level from the full EE profile that extends to the expiration of the longest contract in the netting set. If the original maturity of the longest dated contract contained in the set is greater than 1 year, the effective maturity is calculated according to:

$$M = 1 + \frac{\sum_{t_k > 1yr} EE(t_k)\Delta t_k B(0, t_k)}{\sum_{t_k \leq 1yr} EEE(t_k)\Delta t_k B(0, t_k)},$$

where $B(0, t_k)$ is the risk-free discount factor from the simulation date t_k to today, Δt_k is the difference between time points, $EE(t_k)$ is the expected exposure at time t_k and $EEE(t_k)$ is the effective expected exposure (basically a non-decreasing EE defined in Section 7.2.8). Similar to the general treatment above, M has a cap of 5 years (a 1-year floor is implicitly present in the formula). Note that if the denominator in the above equation becomes rather small then the effective maturity can be large. This means that netting sets with rather small exposure up to 1 year (for example, due to the underlying market value being significantly negative) will have capital determined by a small exposure with a high maturity. For more detail, see Picoult (2005).

For netting sets in which all contracts have an original maturity of less than 1 year, the effective maturity is set to 1 year. However, the 1-year floor does not apply to certain collateralised short-term exposures. The instruments included in this category are OTC derivatives and SFTs that have the original maturity of less than 1 year, are fully or nearly-fully collateralised and subject to daily re-margining. For such transactions, the effective maturity for a given netting set is calculated as the weighted average of the contractual remaining maturities, with notional amounts used as weights.

We show some examples of calculations for M for different exposure profiles in Figure 8.3A. Netting set 1 has a bullet exposure and its effective maturity is therefore slightly smaller than its maturity due to interest rates effects. Due to having a small EE in the first year,[1] netting set 2 has a high effective maturity of 6.51 years, which is capped at 5 years. Finally, netting set 3 has an effective maturity of 3.21 years, which is relatively small since the EE is concentrated within shorter maturities.

[1] This means that the denominator of the formula in Appendix 11.A becomes quite small resulting in the effective maturity being greater than the maximum maturity of the netting set (without the cap of 5 years).

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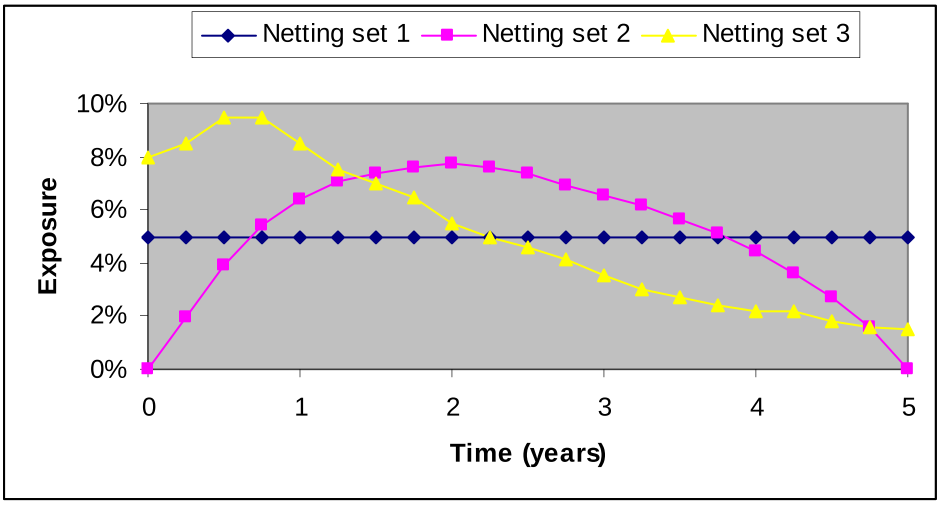

Figure 8.3A. *Illustration of effective maturity for different 5 year EE profiles. Interest rates are assumed to be 5% for all maturities. EE1, EE2 and EE3 have effective maturities of 4.81, 5.00 and 3.21 years respectively.*

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APPENDIX 8B: Double default formula

As noted above, the conditional default probability in the Basel II IRB capital formula is:

$$PD_{99.9\%} = \Phi\left(\frac{\Phi^{-1}(PD_o) + \sqrt{\rho}\Phi^{-1}(99.9\%)}{\sqrt{1-\rho_o}}\right),$$

where it should be noted that PD_o and ρ_o are the default probability and asset correlation parameter of the original counterparty (the obligor). To compute capital for a hedged exposure in the advanced IRB framework (BCBS 2005), it is necessary to calculate the conditional default probability that both the obligor and guarantor will default. It is also important to consider the correlation between obligor and guarantor as high correlations will make the double-default more likely. By assuming an additional asset correlation parameter of ρ_g for the guarantor and an asset correlation between obligor and guarantor of ρ_{og}, the following conditional joint probability formula using a bivariate normal distribution function $\Phi_2(.)$ is used:

$$\Phi_2\left(\frac{\Phi^{-1}(PD_o) + \sqrt{\rho_o}\Phi^{-1}(99.9\%)}{\sqrt{1-\rho_o}}, \frac{\Phi^{-1}\left(PD_g\right) + \sqrt{\rho_g}\Phi^{-1}(99.9\%)}{\sqrt{1-\rho_g}}; \frac{\rho_{og} - \sqrt{\rho_o\rho_g}}{\sqrt{(1-\rho_o)(1-\rho_g)}}\right)$$

A value of of $\rho_{og} = 50\%$ is proposed in order to account for a wrong-way risk due to a correlation between the default probability of obligor and guarantor. Nevertheless, an operational requirement for recognition of double-default is that there is no "excessive correlation" between the credit quality of obligor and guarantor and double-default is not recognised for an exposure to a financial institution. A value of $\rho_g = 70\%$ is used which essentially assumes (conservatively) that the systemic risk of the guarantor is high. This correlation parameter is substantially higher than that for the obligor, ρ_o, which will follow the standard calculation (Appendix 8A) and will therefore be between 12% and 24%. A limiting case of the above formula (for example, as PD_g increases to unity) corresponds to the substitution approach.

The double-default capital formula also includes a loss given default function, which corresponds to the worst case loss when pursuing recoveries from both an obligor and guarantor. Furthermore, the maturity adjustment component will also differ in the event of mismatch between the maturity of the original exposure and that of the protection or guarantee. Any charge for maturity mismatch would be based on the M calculated within the IMM approach.

The Basel Committee have also proposed a simplified approach to the double-default formula where the capital is reduced by the following factor compared to the unhedged exposure case:

$$(0.15 + 160 \times PD_g).$$

The formula was calibrated to the above case and works well for small values of PD_g but can be seen to be more conservative than the unhedged case when $PD_g > 0.531\%$ (this corresponds to the above factor being greater than unity). This is illustrated in Figure 8.4A.

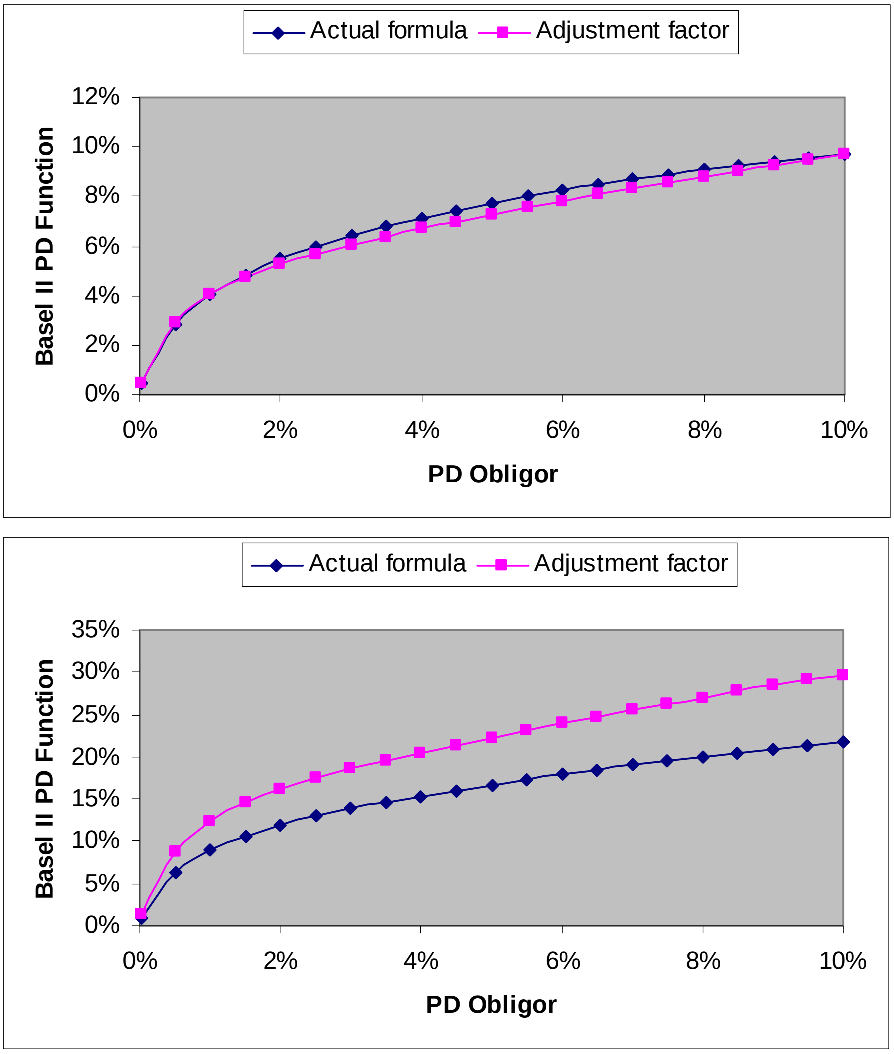

Figure 8.4A. *Double-default formula compared to Basel II adjustment factor for guarantor default probabilities of 0.1% (top) and 0.5% (bottom).*

APPENDIX 8C: The standardised method

The standardised method (SM) in Basel II was designed for banks that are not sophisticated enough for IMM approval but would like to adopt a more risk-sensitive approach than the CEM – for example, to account more properly for netting. Under the SM, one computes the EAD for derivative transactions within a netting set, as a combination of “hedging sets”, which are positions that depend on the same risk factor. Within each hedging set, offsets are fully recognised but netting between hedging sets is not accounted for. As with the CEM, collateral is only accounted for with respect to the current MTM component and future collateral is not specifically considered. The SM is not particularly common (see Figure 8.2 in the book), as banks tend to use the simpler CEM approach or the more sophisticated IMM. Moreover, some regulators have not allowed the standardised approach to be used. The SM defines EAD as:

$$EAD = \beta \times max\left[MTM - C, \sum_i |RPE_i - RPC_i| \times CCF_i\right]$$

Where MTM and C represent the current market value of trades in the netting set and current market value of all collateral positions assigned against the netting set respectively. The terms $|RPE_i - RPC_i|$ represent a net risk position within a “hedging set” i which forms an exposure add-one then multiplied by a conversion factor CCF_i determined by the regulators according to the type of risk position. Finally, β is the supervisory scaling parameter, set at 1.4, which can be considered similar to the alpha factor discussed in Chapter 8.

A hedging set is defined as the portfolio risk positions of the same category (depending on the same risk factor) that arise from transactions within the same netting set. Each currency and issuer will define its own hedging set, within which netting effects are captured. However, netting between hedging sets is not accounted for. Instruments with interest rate and foreign exchange risk will generate risk positions in these hedging sets as well as their own (such as equities or commodities for example). Within each hedging set, offsets are fully recognized; that is, only the net amount of all risk positions within a hedging set is relevant for the exposure amount or EAD. The long positions arising from transactions with linear risk profiles carry a positive sign, while short positions carry a negative sign. The positions with non-linear risk profiles are represented by their delta-equivalent notional values. The exposure amount for a counterparty is then the sum of the exposure amounts or EADs calculated across the netting sets with the counterparty. The use of delta-equivalent notional values for options creates a notable difference compared with the CEM.

As with the CEM, collateral is only accounted for with respect to the current MTM component and future collateral is not specifically considered. The calibration of credit conversion factors (CCFs) is assumed for a 1-year horizon on at-the-money forwards and swaps because the impact of volatility on market risk drivers are more significant for at-the-money trades. Thus, this calibration of CCFs should result in a conservative estimate of PFE. Supervisory CCFs are shown in Table 8.1A.

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Table 8.1A. *Credit conversion factors (CCFs) for financial instrument hedging sets. These are given in paragraphs 86-88 of Annex 4 in BCBS (2006).*

Instrument type	*CCF*
Foreign exchange	2.5%
Gold	5.0%
Equity	7.0%
Precious metals (except gold)	8.5%
Electric power	4.0%
Other commodities (except precious metals)	10.0%

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APPENDIX 8D: Treatment of EAD for repo transactions.

For repo-style transactions, the EAD is calculated as the difference between the market value of the securities and the collateral received, and given by

$$EAD = max[0, MTM(1 + h_s) - C(1 - h_C)],$$

where h_s is the haircut on the security and h_C is the haircut on the collateral. The haircuts must be applied to both the exposure and collateral received in order to account for the risk arising from an appreciation in value of the underlying exposure, and simultaneous decline in value, of collateral received as a result of future market movements. Banks may be permitted to calculate haircuts themselves using internal models. In such cases, the relevant confidence level should be 99% and the minimum time horizon 5 days.

To better account for netting, as an alternative method to the use of haircuts as above, banks may take a VAR-based approach to reflect the price volatility of the exposure and collateral received. Under the VAR-based approach, the EAD or exposure can be calculated, for each netting set, as

$$EAD = \max(0, MTM - C + VAR),$$

where MTM and C again represent the current market value of trades in the netting set and the current market value of all collateral positions held against the netting set, respectively, and VAR represents a value-at-risk type assessment of the collateralised position over some time horizon. For repo-style transactions, the minimum time horizon is five business days (rather than the ten that is standard for OTC derivatives). The advantage of the VAR model is to improve the rule-based aggregation under standard haircuts by taking into account correlation effects between positions in the portfolio. The VAR-based approach is available to banks that have already received approval for the use of internal models under the Market Risk Framework.

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