Counterparty risk in credit derivative contracts

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1.1 Introduction

Counterparty credit risk is the risk that a counterparty in a financial contract will default prior to the expiration of the contract and fail to make future payments. Counterparty risk is taken by each party in an over-the-counter (OTC) derivative and is therefore present in all asset classes, including interest rates, foreign exchange, equity derivatives, commodities and credit derivatives. Given the recent credit crisis and the high profiles failures such as Lehman Brothers, the topic of counterparty risk management has become critically important for many financial institutions and derivatives users. Credit derivatives counterparty risk has been shown to be particularly important and a key driver of some of the financial problems underlying the credit crisis.

CVA (credit value adjustment) is a traditionally applied adjustment to adjust the value of derivative contracts for counterparty risk. CVA accounts for potential future losses due to an institution's counterparties defaulting. The quantification of CVA is therefore an important component in pricing and managing counterparty risk on derivative instruments. Historically, CVA charges have often been incorporated into transactions in favour of the stronger credit quality counterparty. For example, banks trading with corporate counterparties have for many years charged CVAs linked to the credit quality of the corporate and the exposure in question. Recent accountancy rules have also given importance to CVA as a key element in the reporting of accurate earnings information. Accounting standards require an appropriate mark-to-market of derivatives positions including the possibility of future defaults. For example, FASB 157 and IAS39 define fair value and require banks to adjust the risk-free value of derivatives positions the CVA or expected loss associated with future counterparty defaults.

The credit derivatives market has grown dramatically over the last decade, fuelled by the need to transfer credit risk efficiently and develop products that are ever more sophisticated for investors. In the early years of the credit derivative market, counterparty risk concerns were in the back of most people's minds. This, in retrospect, is surprising since the very nature of credit derivative products generates so-called "wrong-way" counterparty risk, a phenomenon arising from an unfavourable relationship between the exposure of a contract and the underlying counterparty default probability. The use of collateral agreements and the perceived credit-worthiness of the large credit derivative dealers were two key reasons why counterparty risk (and indeed wrong-way counterparty risk) was not considered a problem. However, in 2007, the beginnings of the credit crisis crushed such notions and market participants realised the severe nature of counterparty risks in single-name

CDS products and portfolio credit derivatives. A successful future for the credit derivative market is very much linked on the ability to control the inherent counterparty risks.

1.2 Credit value adjustment (CVA)

Credit value adjustment[1] (CVA) is the key component for defining counterparty risk and allows one to express the risky value of a transaction with a given counterparty via:

$$\text{Risky MtM} = \text{Risk free MtM} - \text{CVA} . \tag{1}$$

We should note that CVA is not additive with respect to individual transactions across a "netting set". A netting set defines a group of transactions whose values may be legally netted in the event a counterparty defaults. Netting is a way to mitigate counterparty risk and one or more netting sets may exist for a given counterparty. CVA terms must therefore be computed for each netting set and the risky value of a given transaction cannot be calculated individually as it is defined with respect to other transactions within the same netting set. In this chapter, we will focus on individual transactions and not describe netting effects, which are discussed in more detail by Gregory [2009b].

There have been many models proposed for pricing counterparty risk via CVA, which mostly cover the "classic" instrument types. For example, Sorenson and Bollier [1994], Jarrow and Turnbull [1992, 1995, 1997], Duffie and Huang [1996] and Brigo and Masetti [2005a] describe reduced form models for counterparty risk and focus mainly on interest rate and foreign exchange products. Whilst there is a now a reasonably rich literature on pricing counterparty risk, the discussion of wrong-way CVA such as seen in credit derivative products has also been given only limited coverage. In this chapter, we will explain the nature of counterparty risk in credit derivative products and present quantitative results showing some of the key features.

1.3 Counterparty risk in CDS

A Credit default swap (CDS) is the basic building block of the credit derivative market. CDS, whilst reasonable simple products, have potentially extreme counterparty risks as a direct consequence of their structure. We will start with a discussion and quantification of counterparty risk in the basic CDS product before moving on to consider the more complicated portfolio credit derivative structures.

1.3.1 CDS valuation with no counterparty risk

With no counterparty risk, a CDS can be rather easily defined by the value of the two payment legs corresponding to the premium payments and contingent default

[1] Other names are sometimes used but this seems to be the most common name.

payments (more details can be found in O'Kane [2008]). The present value of the premium leg of a CDS contract is given by: -

$$V_{premium}(t,T) = \sum_{i=1}^{n} S(t,t_i)B(t,t_i)\Delta_{i-1,i}X_{CDS}\,, \tag{2}$$

where n represents the number of remaining premium payments on the CDS, $S(t,t_i)$ represents the risk-neutral survival probability of the reference entity in the period $[t,t_i]$[2], $B(t,t_i)$ represents the risk-free discount factor for time t_i as seen from time t, $\Delta_{i-1,i}$ is the coverage and X_{CDS} is the contractual CDS premium paid on the contract. The default payment made in a CDS contract will occur when the reference entity has defaulted which can occur at any point during the life of the contract. Denoting the reference entity default time by τ, the contingent default payment leg is written as:

$$V_{default}(t,T) = -\mathrm{E}^{Q}\left[(1-\delta)B(t,\tau)I(\tau < T)\right]. \tag{3}$$

Assuming a fixed recovery value[3] of δ, this can be expressed in terms of the survival probability of the reference entity and calculated by discretisation of the resulting integral:

$$V_{default}(t,T) = (1-\delta)\int_{t}^{T} B(t,u)dS(u) \approx (1-\delta)\sum_{i=1}^{m} B(t,t_i)\left[S(t,t_{i-1}) - S(t,t_i)\right]. \tag{4}$$

1.3.2 CDS payoff under counterparty default

A protection buyer in a CDS contract has a payoff with respect to a reference entity's default but is at risk in case the counterparty to the contract suffers a similar fate. The CDS product therefore has a highly asymmetric payoff profile due to being essentially an insurance contract as illustrated in Figure 1.

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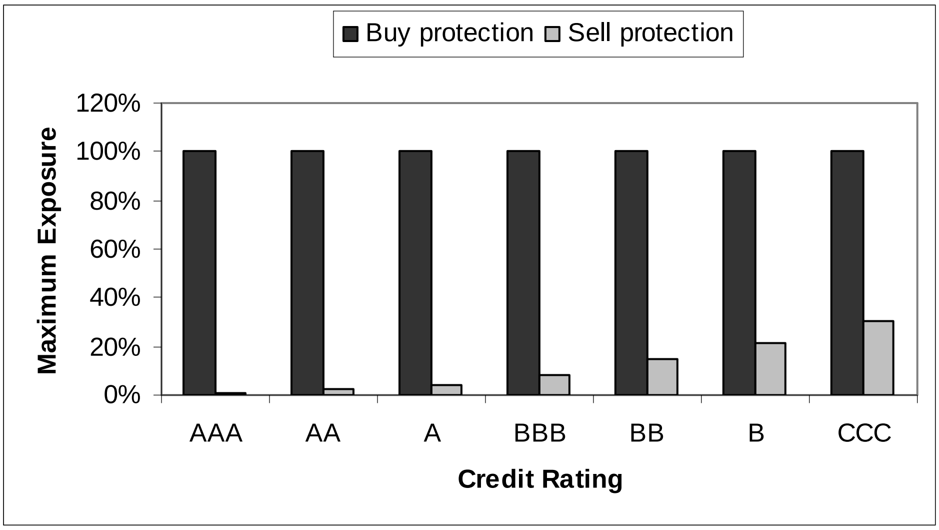
[2] This is the probability of the reference entity not defaulting before time t_i conditional upon not being in default at the current time t.

[3] Or equivalently taking the expected recovery value and assuming independence between recovery value and both the default time and risk-free interest rate.

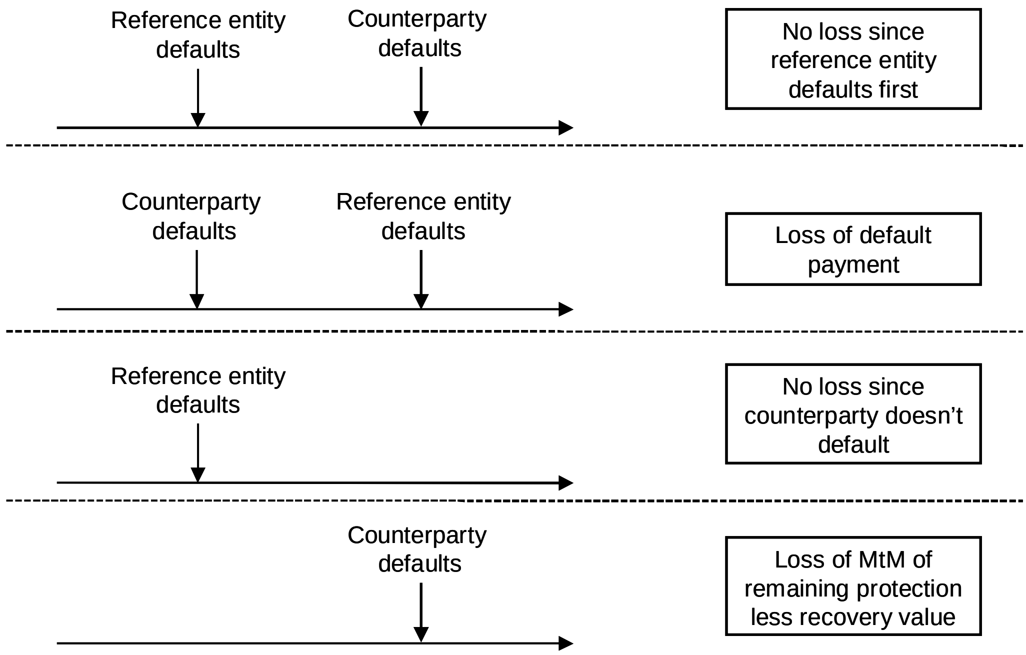
Figure 1. Illustration of the asymmetry of counterparty risk for a CDS. When buying protection, the maximum loss is 100% (reference entity default with zero recovery) but when selling protection it is smaller since it is related only to a tightening of the reference entity CDS premium. We have used ratings as a proxy for credit quality changes and have assumed a 5-year maturity and CDS premiums of 25, 50, 100, 200, 400, 600 and 1000 bps for AAA, AA, A, BBB, BB, B and CCC respectively.



In addition to the asymmetry described above, default correlation is also an important component in defining CDS counterparty risk. Buying CDS protection represents a very definite form of wrong-way risk that increases as the correlation between the credit quality of the reference entity and the counterparty increases. There are four possible cases of relevance when buying protection in a single-name CDS transaction as illustrated in Figure 2: -

- ***Case 1 – Reference entity defaults followed by counterparty***. Here, there is no loss since the reference entity defaults first.
- ***Case 2 – Counterparty defaults followed by reference entity***. Here, there is a significant loss since the counterparty defaults before the reference entity defaults and hence the default payment will not be made.
- ***Case 3 - Reference entity defaults first.*** Here there will be no counterparty risk since the counterparty has not defaulted and the reference entity default will be settled as required.
- ***Case 4 - Counterparty defaults but reference entity does not.*** This is the most complex case. The counterparty defaults and, although the reference entity does not default, any potential positive MtM of the contract will be lost, less some recovery value. If the counterparty default implies a significantly positive MtM on the CDS protection, (since the correlated reference entity is expected to have a worsening credit quality) then this loss would be expected to be significant -- this is the manifestation of wrong-way risk.

Figure 2. Illustration of counterparty risk scenarios for a CDS contract.



1.3.3 Quantifying CVA for a CDS

When calculated the CVA adjustment for CDS, one must account for the default of both the counterparty and the reference entity and, more specifically, the order in which they occur. The pricing requires valuing the two legs of a CDS contingent to the counterparty surviving (since once the counterparty has defaulted an institution would neither make premium payments nor receive default payments) and adding a final term depending on the MtM of the CDS contract at the default time.

We denote by $S^1(t,T)$ the risk-neutral survival probability of both the counterparty and reference entity in the CDS contract. The time t premium payments made in a CDS contract of final maturity T represent an annuity stream with cashflows contingent on joint survival, which can be written as:

$$\tilde{V}_{premium}(t,T) = \sum_{i=1}^{n} S^1(t,t_i) B(t,t_i) \Delta_{i-1,i} X_{CDS}\,, \tag{5}$$

where the other components are as defined for equation (2). The default payment made in a CDS contract will be made when the reference entity has defaulted but only if the counterparty has not previously defaulted. Denoting the counterparty and default time as τ_C and the "first-to-default" time by $\tau^1 = \min(\tau_C, \tau)$, the contingent default payment leg is written as:

$$\tilde{V}_{default}(t,T) = -\mathrm{E}^Q\left[(1-\delta)B(t,\tau)I(\tau^1 < T)I(\tau_C > \tau)\right], \tag{6}$$

where δ is a percentage recovery value for the underlying reference entity, $\mathrm{E}^{Q}[.]$ represents an expectation under the risk-neutral measure and $I(.)$ is an indicator function, which takes the value one if the statement is true and zero otherwise. As for equation (4), equation (6) can be computed via a simple discretisation procedure:

$$\tilde{V}_{default}(t,T) \approx (1-\delta)\sum_{i=1}^{m} B(t,t_i)Q(\tau \in [t_{i-1,}t_i],\tau_C > \tau), \tag{7}$$

where $Q(\tau \in [t_{i-1,}t_i],\tau_C > \tau)$ gives the marginal default probability of the reference entity conditional on survival of the counterparty. This assumes that simultaneous default of counterparty and reference entity is not possible.

Finally, we must add on the payment made at the counterparty default time (case 4 defined previously and illustrated in Figure 2). Denote by $V_{CDS}(\tau,T)$ the (no counterparty risk) MtM or replacement cost of the CDS at some future default date τ including discounting. If this value is positive then the protection buyer will receive only a fraction $\delta V_{CDS}(\tau,T)$ of the amount whilst if it is negative then the MtM must be paid to the defaulted counterparty. Hence the payoff in default is $\delta V_{CDS}(\tau,T)^{+} + V_{CDS}(\tau,T)^{-}$. Finally, we can write the total value of the CDS with counterparty risk as being: -

$$\tilde{V}_{CDS}(t,T) = \tilde{V}_{premium}(t,T) + \tilde{V}_{default}(t,T) + \mathrm{E}^{Q}\left[\left(\delta_C V_{CDS}(\tau,T)^{+} + V_{CDS}(\tau,T)^{-}\right)\right], \tag{8}$$

where δ_C is the counterparty recovery (as opposed to the reference entity recovery). Equation (8) represents the situation from the protection provider's point of view, the protection buyer's position is given simply by reversing the signs on the terms $V_{CDS}(\tau,T)$, $\tilde{V}_{premium}(t,T)$ and $\tilde{V}_{default}(t,T)$.

1.3.4 Modelling approach

We define the random default time of the reference entity via a Gaussian copula by $\tau = S^{-1}(\Phi(Z))$ where $S(t,T)$ represents the survival probability of the reference entity and $\Phi(.)$ denotes the cumulative Gaussian distribution function with Z a standard Gaussian random variable. Then the default time of the counterparty is defined to be correlated and given by $\tau_C = S_C^{-1}(\Phi(Y))$ where $S_C(t,T)$ represents the survival probability of the counterparty. The correlation is introduced by defining $Y = \rho Z + \sqrt{1-\rho^2}\varepsilon$ with ε being an additional independent standard Gaussian random variable with ρ identified as the correlation parameter. The correlation between the reference entity and counterparty default times can also be represented via a bivariate Gaussian distribution. This would mean that the joint survival probability would be given by: -

$$S^1(t,T) = \Phi_{2d}\left[\Phi^{-1}(S(t,T)),\Phi^{-1}(S_C(t,T);\rho)\right], \tag{9}$$

where $\Phi^{-1}(.)$ is the inverse of a cumulative Gaussian distribution function and $\Phi_{2d}(.)$ represents a cumulative bivariate Gaussian distribution function with correlation parameter ρ. The marginal default probability term can be approximated by:

$$\begin{aligned} &Q(\tau \in [t_{i-1,} t_i], \tau_C > \tau) \approx Q(\tau > t_{i-1}, \tau_C > t_i) - Q(\tau > t_i, \tau_C > t_i) \\ &= \Phi_{2d}\left[\Phi^{-1}\left(S(t, t_{i-1})\right), \Phi^{-1}\left(S_C(t, t_i)\right); \rho\right] - \Phi_{2d}\left[\Phi^{-1}\left(S(t, t_i)\right), \Phi^{-1}\left(S_C(t, t_i)\right); \rho\right] \end{aligned}, \quad (10)$$

which will be accurate for small time intervals where the probability of both reference entity and counterparty defaulting within the interval is negligible. The contingent premium and default terms, $\tilde{V}_{premium}(t,T)$ and $\tilde{V}_{default}(t,T)$, can then be computed analytically in from equations (6) and (7) using the expressions in equation (9) and (10). The number of points used in equation (7) needs to be reasonably large (at least 20 per year), especially when correlation is high.

The computation of the last term in equation (8) is more complicated since it involves the replacement cost corresponding to the risk-free value of the CDS at some future date τ_C. Furthermore, the option like payoff of this term means that not only the expected value of the CDS is required but also the distribution of future CDS value at the counterparty default time. Whilst the expected value of the CDS at the default time can be calculated in the static copula approach, the optionality inherent in the counterparty risk calculation requires the use of a dynamic credit model. Furthermore, the computation of this replacement cost involves a classic American Monte Carlo problem. More complex models are described, for example, by Brigo and Capponi [2009] and Lipton and Sepp [2009]. We will take a more simple pricing approach based on the fact that, as pointed out by Mashal and Naldi [2005], upper and lower bounds for the final term in equation (8) can be defined by:

$$\delta.\mathrm{E}^Q\left[V_{CDS}(\tau_C, T)\right]^+ + \mathrm{E}^Q\left[V_{CDS}(\tau_C, T)\right]^-, \qquad \text{(upper bound)} \qquad (11a)$$

$$\mathrm{E}^Q\left[\left(\delta C_{CDS}(\tau_C, T)^+ + C_{CDS}(\tau_C, T)^-\right)\right], \qquad \text{(lower bound)} \qquad (11b)$$

where $C_{CDS}(\tau_C, T)$ represents the value of the cashflows in the CDS contract at time τ_C in a given scenario, discounted back to today. The upper and lower bounds defined by the above equation can be computing by Monte Carlo simulation directly as discussed also by Turnbull [2005]. It is possible to compute the upper bound analytically since we can use the results of Laurent and Gregory [2005] to calculate the survival probability of the reference entity conditional upon the counterparty default:

$$Q(\tau \geq t_2 \mid \tau_C = t_1) = \frac{\int_{-\infty}^{\infty} \Phi\left(\frac{\sqrt{\rho(1-\rho)}u - \rho\Phi^{-1}\left(S_C(t_1)\right) + \Phi^{-1}\left(S(t_1)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du}{\int_{-\infty}^{\infty} \Phi\left(\frac{\sqrt{\rho(1-\rho)}u - \rho\Phi^{-1}\left(S_C(t_1)\right) + \Phi^{-1}\left(S(t_2)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du}. \qquad (12)$$

The above term is rather conveniently calculated via standard quadrature methods. The conditional survival function above allows us to calculate the expected value of the CDS contract at the counterparty default time as required by the term $\mathrm{E}^Q\left[V_{CDS}(\tau_C,T)\right]$ in equation (9).

We will use both the Monte Carlo and analytical approaches to calculate the fair CDS premium in the presence of counterparty risk. We note a final complexity, which is that, since the term $V_{CDS}(\tau_C,T)$ depends on the premium itself, we need to solve recursively for this premium. In practice, due to the relatively linearity in the region of the solution, the convergence is almost immediate.

We note that the above expressions describe the risky MtM of a CDS without explicit reference to a CVA term. Given the wrong-way risk inherent in the product, this is more rigorous and easier to understand. The CVA could be computed simply by comparison to the risk-free MtM value as indicated by equation (1).

We will also ignore the impact of any collateral in the following analysis. This will be conservative since the use of collateral may be considered to reduce significantly CDS counterparty risk. However, due to the highly contagious and systemic nature of CDS risks, the impact of collateral may be hard to assess and indeed may be quite limited, especially in cases of high correlation. We note also that many protection sellers in the CDS market (such as monolines) have not traditionally entered into collateral arrangements anyway.

1.3.5 Parameters

In order to compute the risky value of buying CDS protection as a function of correlation between the reference entity and counterparty (the counterparty is selling protection in the base case). We assume the following base case parameters: -

$h = 2\%$	Hazard rate of reference entity.
$h_C = 4\%$	Hazard rate of counterparty.
$\delta = 40\%$	Recovery rate for reference entity.
$\delta_C = 40\%$	Recovery rate for counterparty.
$T = 5$	Maturity of CDS contract.

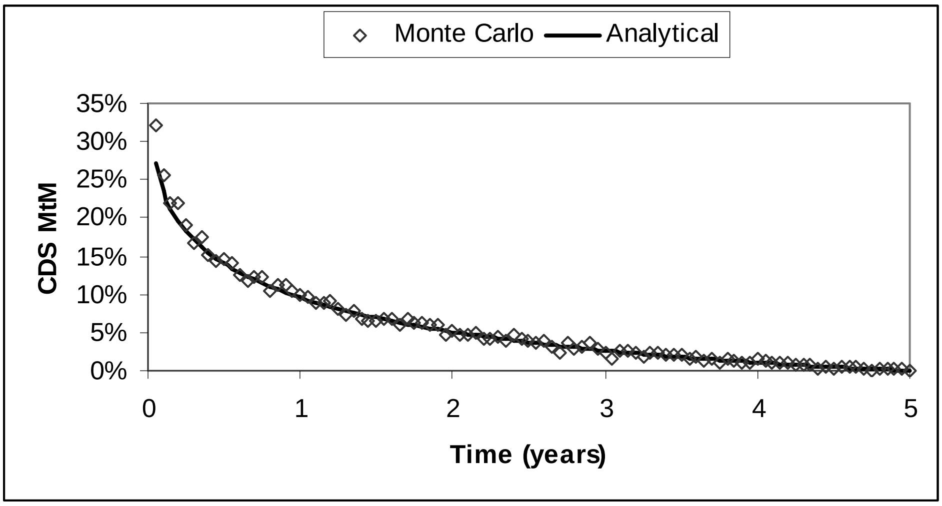
The survival probabilities of reference entity and counterparty are defined by the hazard rates according to $S(t,u) = \exp[-h(u-t)]$ and $S_C(t,u) = \exp[-h_C(u-t)]$. We can calculate the approximate CDS premiums for reference entity and counterparty from $X_{CDS} \approx h(1-\delta)$ which gives 240 and 120 basis points per annum[4]. This assumes approximately equal CDS premiums for all maturities. It is possible to lift this assumption and calibrate $S(.)$ and $S_C(.)$ to a term structure of default probability without significant additional complexity.

[4] The calculations used hazard rates to give precisely these CDS premiums.

1.3.6 CDS replacement cost

We begin by calculating the long protection CDS replacement cost at the counterparty default time that defines the final term in equation (8). If positive, this term will relate to a recovery amount of the CDS MtM at the counterparty default time. If negative then it corresponds to an amount owed to the defaulted counterparty. For the example concerned, the CDS replacement cost is shown in Figure 3 for a correlation parameter of 50%. It is interesting to quantify the expected MtM of the CDS at the counterparty default time. With positive correlation, counterparty default represents “bad news” for the reference entity credit quality and hence the long protection CDS is expected to have a positive value. The sooner the counterparty default time, the more significant this impact[5].

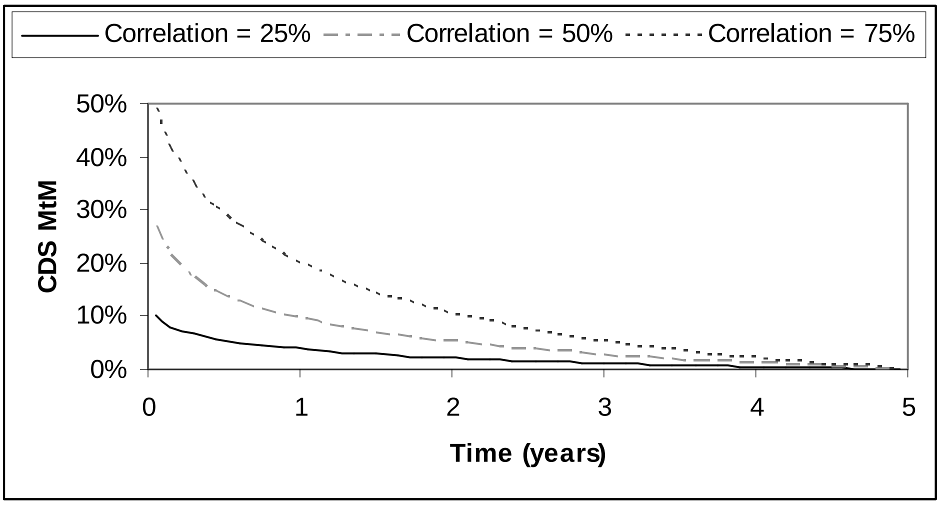
Figure 3. Expected long protection CDS MtM value (replacement cost) at the counterparty default time computed using analytical and Monte Carlo approaches for an assumed correlation of 50% between the counterparty and reference entity. The Monte Carlo results use 1,000,000 simulations with the calculations bucketed with a width of 0.05 years.



In Figure 4, we show the expected CDS MtM at the counterparty default time as a function of correlation. Higher correlation has a more significant on the expected value of the long protection CDS contract since the reference entity credit quality is expected to be significantly worse at the counterparty default time. This suggests that at high correlation the upper bound may be close to the actual result since there is little chance that the long protection position can have negative value meaning that the first term in equation (11a) will dominate and hence the last term in equation (8) will be well approximated by ignoring the negative contribution. Put differently, the option payoff with respect to the replacement CDS is very in-the-money and hence the impact of volatility should be small.

[5] At some point, the counterparty default becomes no longer “bad news” as the default is expected. In this example, the expected counterparty default time is 25 years (the inverse of the hazard rate) and hence within 5 years is rather unexpected and has a significant impact on the expected value of the CDS contract.

Figure 4. Expected long protection CDS MtM value (replacement cost) at the counterparty default time as a function of correlation computed analytically.

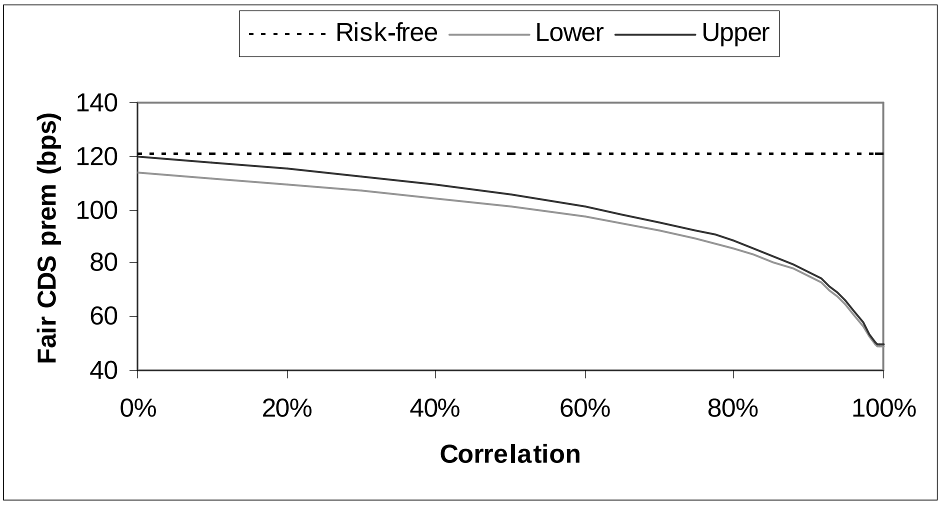


1.3.7 Buying CDS protection

We start by considering the fair premium (i.e. reduced in order to account for counterparty risk) that one should pay in order to buy protection, which is shown in Figure 5. Firstly, we see that the upper and lower bounds are quite close, making a more costly computation of the exact result less important. Furthermore, the upper and lower bounds converge at high correlation which can be understood by the previous impact of correlation on CDS replacement value in Figure 4. We can also observe the very strong impact of correlation: one should be willing only to pay 100 bps at 60% correlation to buy protection compared with paying 120 bps with a "risk-free" counterparty. The CVA in this case is effectively 20 bps or one sixth of the risk-free CDS premium. At extremely high correlations, that the impact is even more severe and the CVA adjustment can be seen to be huge. At a maximum correlation of 100%, the CDS premium is just above 48 bps, which relates entirely to the recovery value[6]. A long protection CDS contract has an increasing CVA as correlation increases due to wrong-way risk.

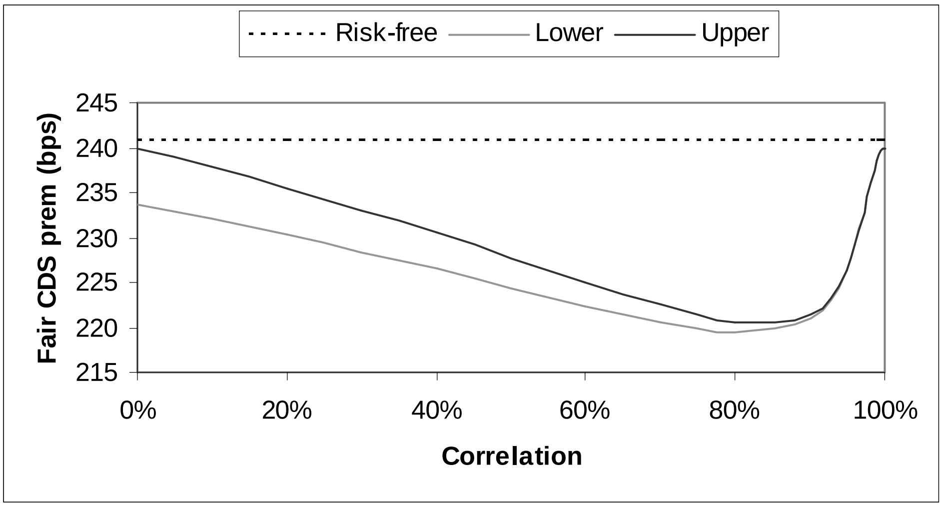
[6] The premium based only on recovery value, i.e. the protection buyer will always receive the recovery fraction times the MtM of the contract at the counterparty default time and there is no chance of receiving any default payment, is 120 × 40% = 48 bps.

Figure 5. Upper and lower bounds for the fair CDS premium when buying protection subject to counterparty risk compared to the standard (risk-free) premium.



In Figure 6, we show the same example but with the hazard rates of the reference entity and counterparty exchanged. We can notice that the contract does not contain as much counterparty risk since the protection seller has a better credit quality than the reference entity. We also observe that the counterparty risk vanishes as the correlation goes to 100%. This is due to the fact that, with perfect correlation, the more risky reference entity will always default first. This facet might be considered slightly unnatural. An obvious way to correct for it would be to have some concept of joint default of the reference entity and counterparty or build in a settlement period to the analysis. These points are discussed respectively by Gregory [2009a] and Turnbull [2005].

Figure 6. As previous figure but with the hazard rates of the reference entity and counterparty swapped.



1.3.8 Selling CDS Protection

We now consider the impact of selling CDS protection to a risky counterparty and use the same base case parameters as in the previous section. In Figure 7 and Figure 8, we show the fair CDS premiums (increased to account for counterparty risk). We ignore the impact of negative correlations, which are highly unlikely in practice due to the correlation inherent in credit markets. The use of upper and lower bounds is not as useful as in the long protection case. For zero or low correlation values, the lower bound is more relevant since the protection seller may possibly suffer losses due to the counterparty defaulting when the CDS has a positive MtM (requiring a somewhat unlikely tightening of the reference entity credit spread). However, for high correlation values, the upper bound is more relevant since the short CDS replacement cost is very likely to be negative, meaning that there is no recovery value according to equation (11a) and virtually no counterparty risk. A short protection CDS contract has a decreasing CVA as correlation increases due to right-way risk.

Figure 7. Upper and lower bounds for the fair CDS premium when selling protection subject to counterparty risk compared to the standard (risk-free) premium.

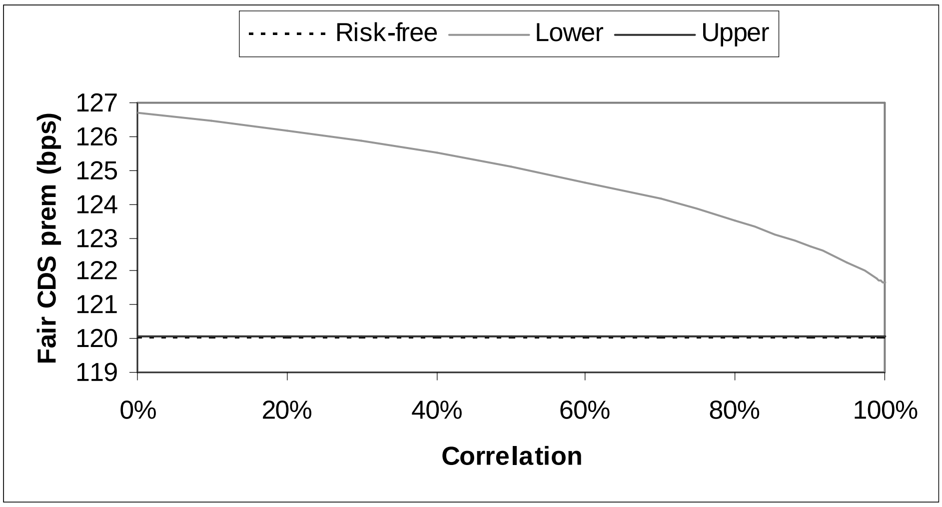
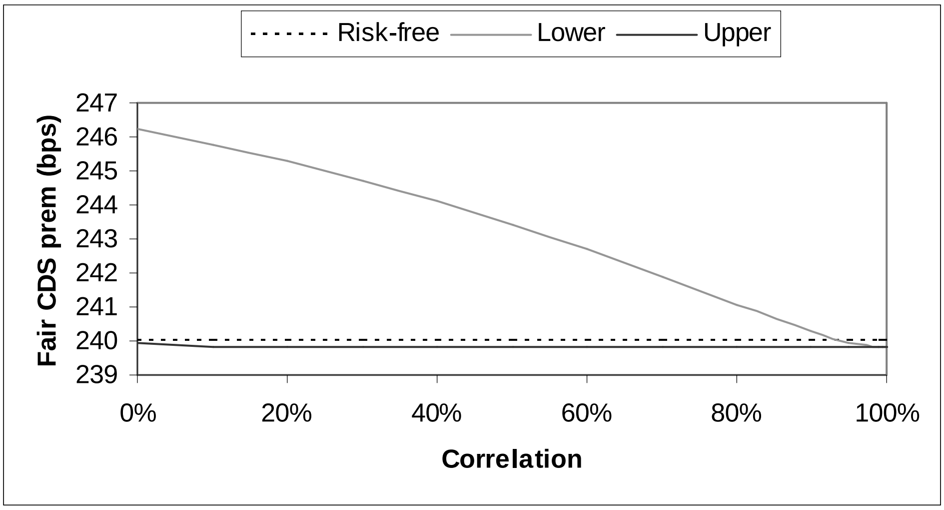


Figure 8. As previous figure but with the hazard rates of the reference entity and counterparty swapped.



1.3.9 Bilateral CDS counterparty risk

A trend that has become increasingly relevant and popular recently has been to consider the bilateral nature of counterparty risk meaning that an institution would evaluate counterparty risk under the assumption that they, as well as their counterparty, may default. This is done on the basis that a defaulting institution "gains" on any outstanding liabilities that need not (cannot) be paid in full. This component is often named DVA (debt value adjustment). DVA is becoming commonly accepted by market participants and indeed is allowed under accountancy regulations. Many institutions regard bilateral considerations as important in order to agree on new transactions, unwinds and minimise PnL volatility.

In the last few years, many institutions have included their own default probability when quantifying counterparty risk. The use of DVA is somewhat controversial (e.g. see Gregory [2009a]). However, when considering wrong-way risk products such as credit derivatives, bilateral counterparty risk is less of an issue. The two terms (CVA and DVA) will likely be linked to either wrong or right-way risk. Wrong-way risk will have the impact of increasing either CVA (DVA)[7] whilst right-way risk will correspondingly decrease DVA (CVA). This then removes some of the complexity of bilateral counterparty risk and creates a situation closer to the unilateral treatment. To evaluate CDS counterparty risk, the protection seller's default probability is the main component to consider, with the protection buyer's default having only secondary importance. In terms of agreeing on transaction counterparty risk charges, a protection seller would probably have to agree to the pricing of a long CDS position as shown in section 1.3.6.

It is possible to do the above calculations under the assumptions that both counterparties may default as described by Turnbull [2005]. However, this has a limited impact on the calculations since the counterparty risk all resides with the protection buyer in the contract. Hence, the DVA component from the protection buyer's point of view will simply be reduced by a small amount due to the possibility that they may default first. Other than that, the conclusions are similar.

1.4 Counterparty risk in structured credit products

Whilst CDS counterparty risk represents a challenge to quantify due to the wrong-way risk and uncertainty of the correlation between reference entity and protection seller (or buyer), structured credit has given rise to even more complex counterparty risk in the form of tranches. There exist many kinds of so-called CDO (collateralised debt obligation) structures, which are all broadly characterised by their exposure to a certain range of losses on a portfolio. The counterparty risk problem now becomes more complex since one needs to assess where the counterparty might default compared to all the reference names underlying the portfolio. Our discussion will consider index tranches, the most commonly traded portfolio credit derivatives. However, the general conclusions will hold for all CDO products.

1.4.1 Index tranches

[7] Assuming there is not a substantial difference between the impact of the counterparty and institution defaults on the exposure distribution.

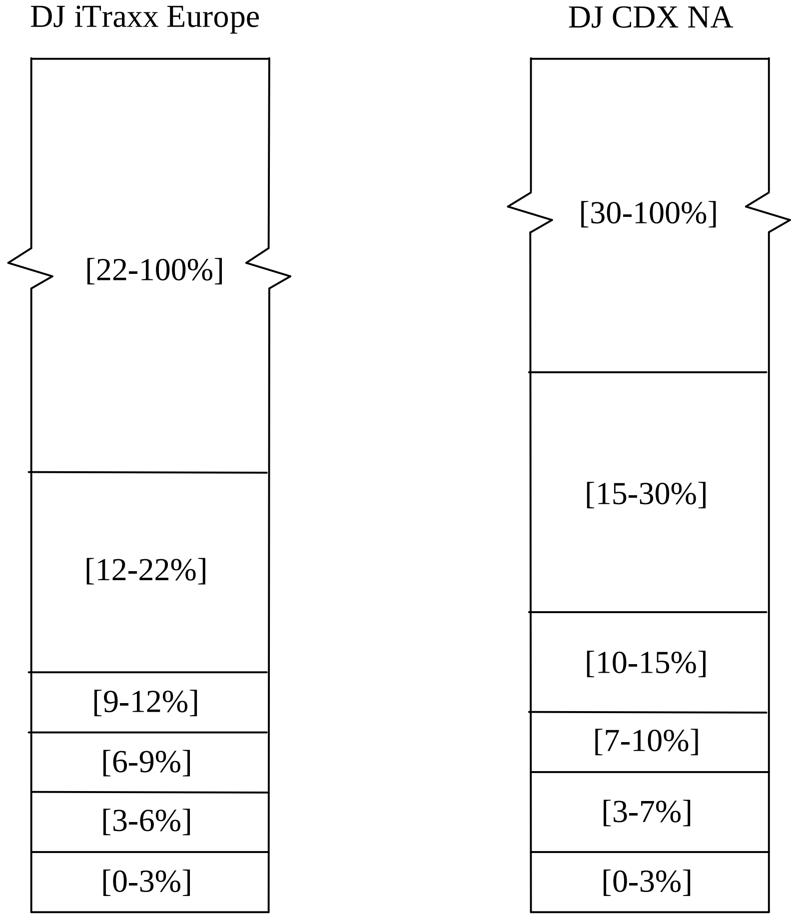
Credit indices represent the most liquid forms of traded credit risk. A credit index can be thought of as an equally weighted combination of single-name CDS and the fair premium on the index will be close to the average CDS premium within that index[8]. The two most common credit indices are:

- **DJ iTraxx Europe.** This contains 125 European corporate investment grade reference entities, which are equally weighted.
- **DJ CDX NA IG.** This contains 125 North American (NA) corporate investment grade reference entities that are also equally weighted.

Other indices exist for different underlying reference entities and regions but they are less liquid. Buying CDS protection on $125m of the DJ CDX NA IG index (for example) is almost[9] equivalent to buying $1m of CDS protection on each of the underlying reference entities within the index. Whilst a credit index references all losses on the underlying names, a tranche will only reference a certain portion of those losses. So for example, an [X, Y%] tranche will reference losses between X% and Y% on the underlying index. The "subordination" of the tranche is X% whilst Y% is referred to as the "detachment point". The size of the tranche is (Y - X)%. The standard index tranches for the DJ iTraxx Europe and DJ CDX NA indices are illustrated in Figure 9. The [0-3%] equity tranches are the most risky instruments since they are completely exposed to the first few defaults on the portfolio. As one moves up through the "capital structure", the tranches becomes less risky.

[8] This is not quite true for two reasons. Firstly, a theoretical adjustment must be made to the average CDS premium to account for the heterogeneity of the constituents. Secondly, the index will typically trade at a basis to the average CDS premiums (bid-offer costs will prevent arbitrage of this difference).
[9] Aside from the theoretical adjustment due to a premium mismatch and the fact that the index protection may involve an up-front payment.

Figure 9. Illustration of the index tranches corresponding to the DJ iTraxx Europe and DJ CDX North American credit indices. All tranches are shown to scale except the [22-100%] and [30-100%].



Equity tranches ([0-3%]) have always traded with an up-front premium and fixed running spread of 500 bps to avoid the annuity risk that exists for such a relatively high risk tranche. For iTraxx, more recently the [3-6%] and [6-9%] have changed to trade in the same way. The remaining tranches trade on a running basis. CDX tranches (which used to trade in a similar way to iTraxx) now trade at 500 basis points (bps) running for [0-3%], [3-7%] and [7-10%] and 100 basis points running for [10-15%], [15-30%], and [30-100%]. Example tranche quotes for iTraxx and CDX are shown in Table 1.

Table 1. Example tranche quotes for iTraxx and CDX investment grade tranches at 5, 7 and 10-year maturities for 28th August 2009. The first three tranches in each case trade with a fixed 500 bps running coupon with the quote reflecting the up-front payment required. The final three iTraxx tranches trade with a variable coupon (shown in bps) only whilst the final three CDX tranches trade at a fixed running coupon of 100 bps with the quote reflecting the up-front payment required. Up-front payments are negative when the fixed coupon is higher than the fair coupon.

DJ iTraxx Europe Tranches			
	5Y	7Y	10Y
[0-3%]	38.00%	45.00%	50.75%
[3-6%]	2.000%	7.000%	12.750%
[6-9%]	-7.625%	-6.000%	-2.875%
[9-12%]	160	200	246
[12-22%]	66.5	92.0	101.5
[22-100%]	28.75	34.00	38.25

DJ CDX NA			
	5Y	7Y	10Y
[0-3%]	70.125%	76.250%	77.875%
[3-7%]	24.500%	32.375%	36.875%
[7-10%]	1.125%	6.000%	10.750%
[10-15%]	4.500%	9.438%	13.625%
[15-30%]	-1.280%	-1.150%	1.030%
[30-100%]	-2.300%	-3.400%	-4.800%

Irrespective of trading convention, the important aspect of an index tranche is that it covers only a certain range of the losses on the portfolio. Index tranches vary substantially in the risk they constitute: equity tranches carry large amount of risk and pay attractive returns whilst tranches that are more senior have far less risk but pay only moderate returns. At the far end, super senior tranches ([22-100%] and [30-100%]) might be considered to have no risk whatsoever (in terms of experiencing losses). Tranching creates a leverage effect since the more junior tranches carry more risk than the index whilst the most senior tranches[10] have less risk. For ease of comparison, the results below will assume that all tranches trade on a fully running basis to ease the comparison across the capital structure. Whilst tranches have different trading conventions, as noted above, this does not influence the results substantially as shown by Gregory [2009b].

The goal is to understand the impact of counterparty risk for index tranches or CDO products traded in unfunded form. It is possible to extend the analysis of the previous section to calculate the upper and lower bounds on the value of a tranche product in the presence of counterparty risk. More details on this can be found in Turnbull [2005] and Pugachevsky [2005]. Our calculations follow these authors, although we will again calculate the fair premiums for risky tranche instruments, which are probably the easiest numbers to illustrate the impact of counterparty risk.

[10] Due to its size, usually only the super senior may have a leverage of less than one and all other tranches may be more highly leveraged than the index.

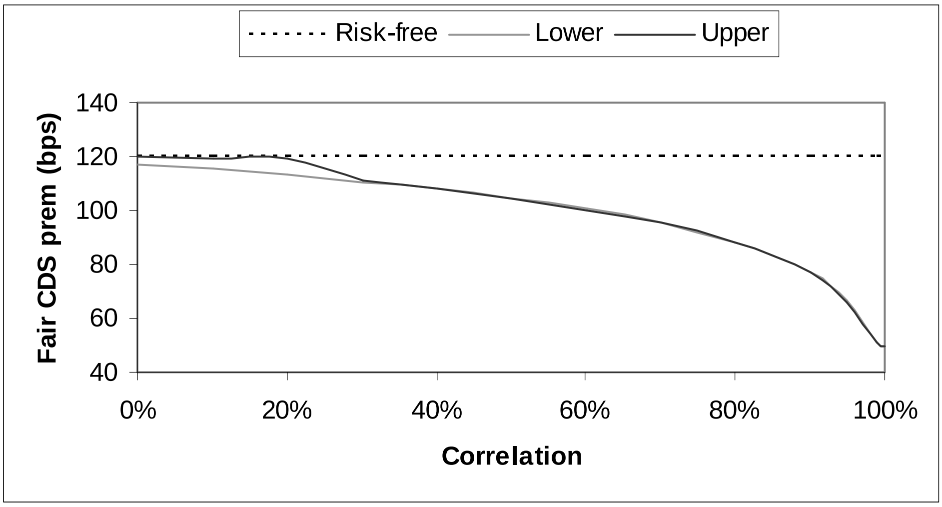
The following parameters will be used in the examples: -

$n = 125$	Number of reference entities within the portfolio (consistent with iTraxx).
$\bar{h} = 2\%$	Average hazard rate of a name in the portfolio[11].
$h_C = 4\%$	Hazard rate of counterparty.
$\delta = 40\%$	Recovery rate of reference entity.
$\delta_C = 40\%$	Recovery rate of counterparty.
$T = 5$	Maturity of CDS contract.

1.4.2 Credit indices and counterparty risk

We first compute the fair CDS premium when buying protection on a CDS index. In Figure 10, we show the fair CDS premium upper and lower bounds compared to the risk-free value. We see almost exactly the same result as seen previously for a single name CDS with equivalent parameters in Figure 5. Hence we can conclude that a credit index behaves in a very similar way to a similar single-name CDS in terms of counterparty risk.

Figure 10. Upper and lower bounds for the fair CDS premium when buying protection on a CDS index subject to counterparty risk compared to the standard (risk-free) premium.



1.4.3 Index Tranches and counterparty risk

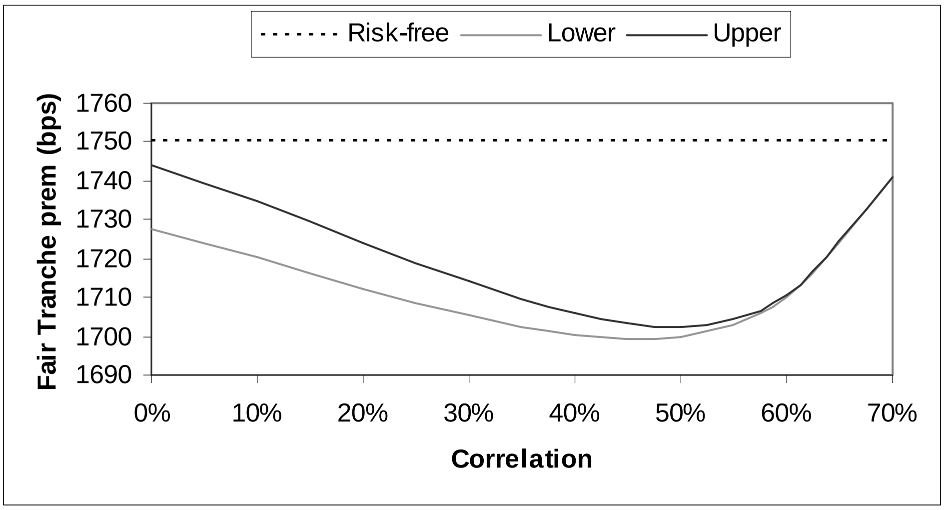
For tranches of a portfolio, it is important to understand how the impact of counterparty risk can change across the capital structure. As mentioned previously, we choose tranches according to the standard iTraxx Europe portfolio that are defined by the attachment and detachment points [0%, 3%, 6%, 9%, 12%, 22%, 100%]. Since

[11] All of the following results have been computed with both homogeneous and heterogeneous hazard rates. There were no significant qualitative differences in the results and so for easy of replication of results we show the former results. We also note that the precise hazard rate was chosen to as to give a fair price for the index of 120 bps.

we are interested only in understanding the qualitative impact of counterparty risk for different tranches, we choose the market standard Gaussian copula model (see chapter ?) with a fixed correlation parameter of 50%[12]. Due to constraints on the correlation matrix, this means we consider the correlation between the counterparty default and the other names in the portfolio in the range [0, 70%][13].

We first show the fair premium for buying [0-3%] protection (Figure 11) and can see that the counterparty risk impact is actually quite small, even at high correlation values. At the 40% recovery rate assumed, the equity tranche covers the first 6.25 defaults[14] in the portfolio. Even though the counterparty is more risky, the chance that it defaults at some point before the equity tranche has completely defaulted is relatively small[15]. The impact of correlation (between counterparty default and the reference names in the portfolio) is quite subtle. As correlation increases, the counterparty risk at first increases also (decreasing fair premium) due to the more risky counterparty being more likely to default earlier. However, for very high correlations, we see the effect reversing which is due to approaching the maximum correlation allowed which makes the counterparty default time increasingly certain vis à vie the other defaults[16].

Figure 11. Upper and lower bounds for the fair premium when buying protection on the [0-3%] equity tranche (assuming the premium is paid on a running basis) as a function of correlation with the parameters given in the text.



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[12] This does not produce prices close to the market but the standard approach of "base correlation" used to reproduce market prices does not have an obvious associated way in which to price correctly counterparty risk. We have checked that the qualitative conclusions of these results hold at different correlations levels.

[13] The upper limit for this correlation due to constraints of positive semi-definitiveness on the correlation matrix is approximately $\sqrt{50\%} = 70.7\%$.

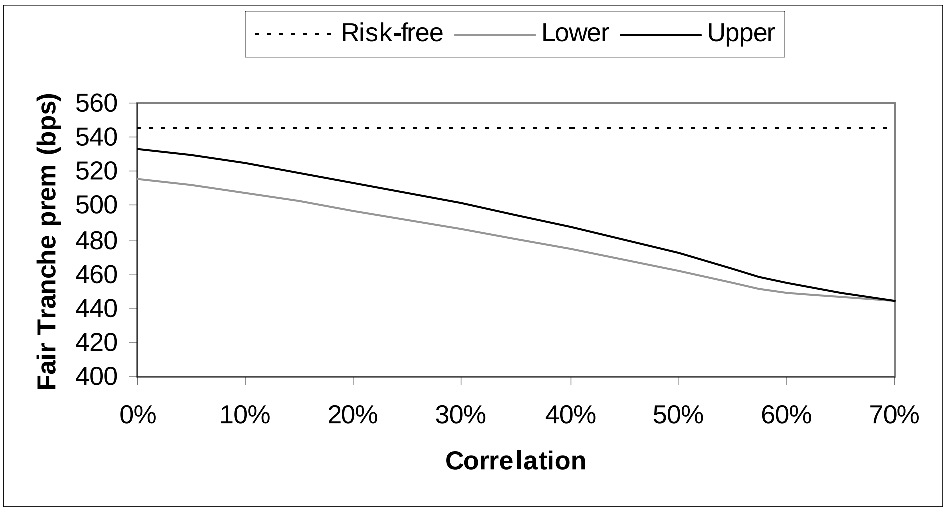
[14] 3% × 125 / (1-40%).

[15] The counterparty must be one of the first seven defaults for their to be any counterparty risk since after this point the tranche is completely wiped out.

[16] This is a subtle point relating to the order of default times at high correlation. Due to the relative riskiness of the counterparty with respect to the other names and the correlation structure, the counterparty default is expected to be early but unlikely to be within the first seven defaults and hence he equity tranche has little counterparty risk.

We now look at a significantly more senior part of the capital structure with the [6-9%] tranche in Figure 12. We can see that the counterparty risk is much more significant, and increases substantially with the correlation between the counterparty and reference entities in the portfolio. At high correlation, the fair risky premium is decreased by around 100 bps compared to the risk-free premium. The impact of increasing correlation can again be understood by increasing the likelihood that the more risky counterparty will default sooner rather than later. Since the [6-9%] tranche is only hit after 12.5 defaults, there is more chance that the counterparty will have defaulted prior (or during) the tranche taking losses.

Figure 12. Upper and lower bounds for the fair premium when buying protection on the [6-9%] tranche as a function of correlation with the parameters given in the text.



1.4.4 Super senior risk

Super senior tranches have created a big headache for the credit market in terms of their counterparty risk. Let us start by asking ourselves how many defaults would cause a loss on a super senior tranche of DJ iTraxx. We can represent the number of default a given tranche can withstand as: -

$$\text{Num Defaults} = n\frac{X}{(1-\bar{\delta})}, \tag{13}$$

where X represents the attachment point of the tranche in percent, n is the number of names in the index and $\bar{\delta}$ is the (weighted[17]) average recovery rate for the defaults that occur.

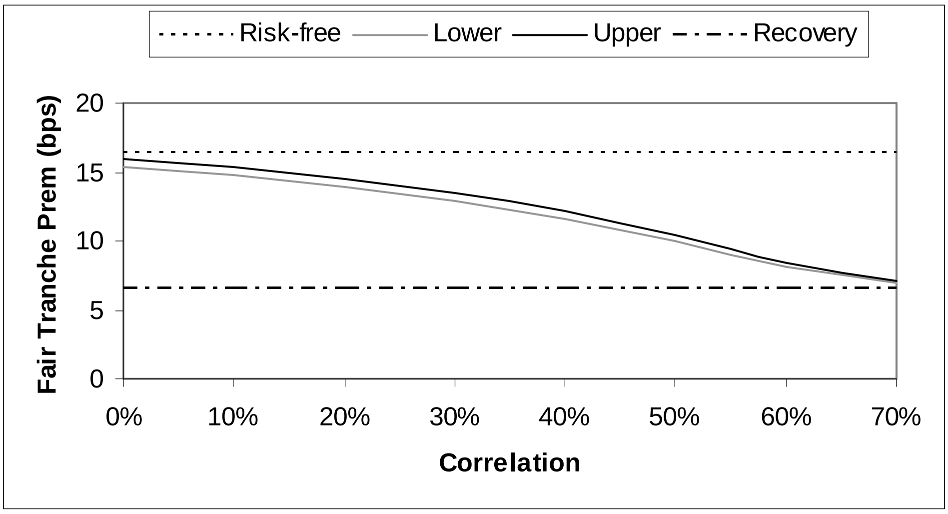
Super senior tranches clearly have very little default risk. Even assuming (conservatively) zero recovery, default rates over several years would have to be many multiples[18] of historical averages to wipe out the subordination on the super senior tranches. This default remoteness has let to terms such as “super triple-A” or

[17] Since the default that actually hits the tranche may have only a fractional impact as in the previous example.

[18] For example, Gregory [2009b] estimates default rates of 4 to 6 times for a 5-year [22-100%] tranche and 8 to 11 times for a 10-year tranche.

"quadruple A" being used to describe the risk on super senior tranche since they constitute what we might call "end of the world" risk.
Finally, we consider the most senior tranche in the capital structure, the super senior [22-100%] in Figure 13. Assuming 40% recovery, there need to be 45.8 defaults[19] before this tranche takes any loss and so the chance that the counterparty is still around to honour these payments is expected to be much smaller than for other tranches. Not surprisingly, the counterparty risk impact is now dramatic with the fair premium tending towards just a recovery value at high correlation (40% of the risk-free premium). In such a case there is virtually no chance to settle losses on the protection before the counterparty has defaulted. We could argue that a more appropriate recovery rate would be close to zero (since an institution selling protection on super senior positions is likely to be highly leveraged as in the case of monolines). This would of course mean that the protection could have little or no value at high correlation.

Figure 13. Upper and lower bounds for the fair premium when buying protection on the [22-100%] super senior tranche as a function of correlation with the parameters given in the text. The fair premium based on a recovery only assumption is shown – this assumes the counterparty will never settle any losses before defaulting.

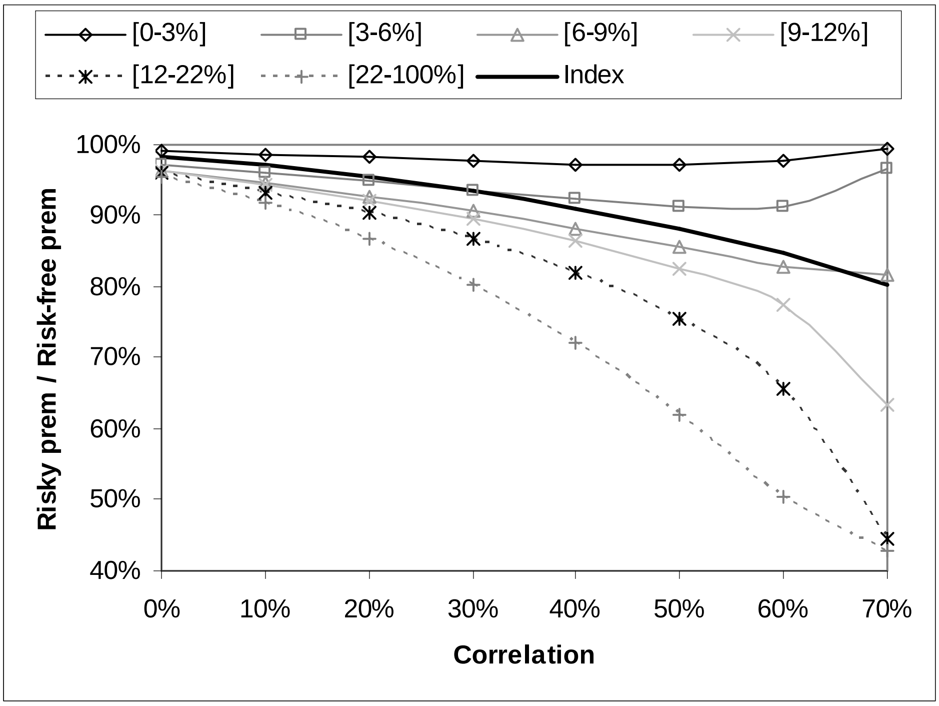


1.4.5 Counterparty risk distribution across capital structure

We summarize the above results by showing the impact of counterparty risk across the entire capital structure in Figure 14. In order to compare all tranches on the same scale, we plot the ratio of fair risky premium (as an average of the upper and lower bounds) to the risk-free premium: this value will have a maximum at unity and decrease towards the recovery (of the counterparty) as counterparty risk becomes more significant. Whereas, the equity tranche has less risk than the index, all other more senior tranches have more risk (except the [3-6%] tranche at most correlation levels). Indeed, from a counterparty risk perspective, we can view tranching as segregating the counterparty risk – the more senior a tranche, the more risk it contains on a relative basis.

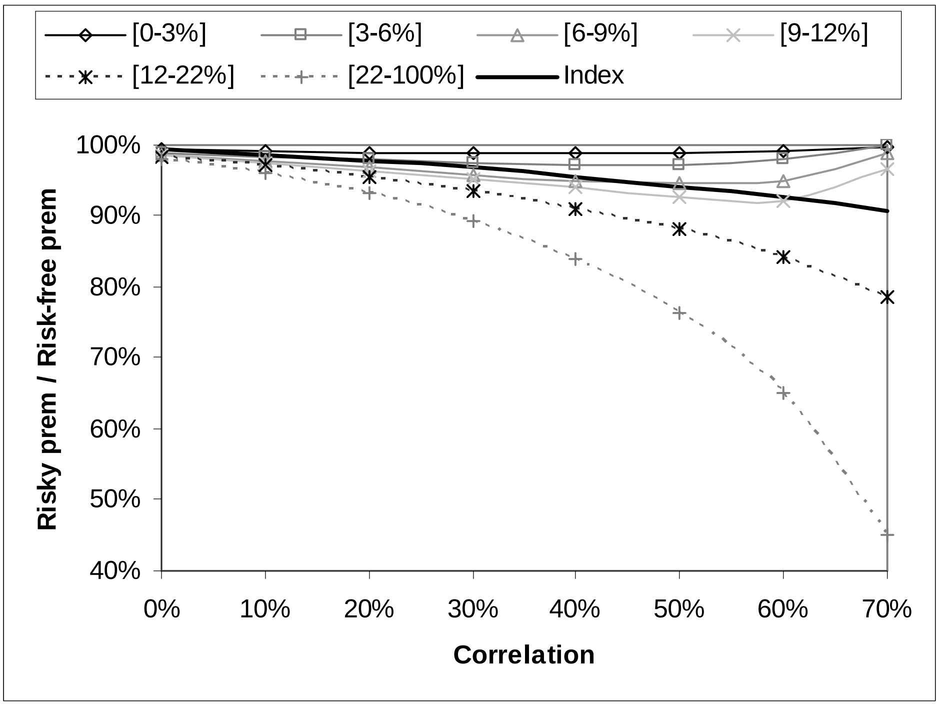
[19] $22\% \times 125/(1-40\%)$

Figure 14. Impact of counterparty risk across the capital structure. Fair risky tranche premium divided by the risk-free premium for all tranches in the capital structure and compared to the index ([0-100%] tranche).



The above analysis concerned a situation where the counterparty is more risky than the average of the portfolio. We briefly summarize results for a less risky counterparty with a hazard rate of $h_C = 1.5\%$ in Figure 15. Whilst the overall impact is, as expected, not so significant we still see that there is still considerable counterparty risk, especially for the most senior tranches.

Figure 15. As previous figure but for a less risky counterparty with $h_C = 1.5\%$.



The fact that counterparty risk increases with the seniority of the tranche, is an important aspect of portfolio credit products. We can also note from the above figure that the extreme counterparty risk of the [22-100%] tranche is not significantly decreased from trading with the counterparty that is two and a half times less risky. Very importantly, we see that the seniority of a tranche can dominate over even the credit quality of the counterparty. This was an important lesson in some of the problems banks had in buying super senior protection from monoline insurers (see Gregory [2008b]).

1.5 Summary

In this chapter, we have described the counterparty risk of credit derivative instruments. We have discussed the so-called "wrong-way risk" in credit derivatives products and how this has been a significant issue for controlling their counterparty risk. We have quantified counterparty risk on single name credit default swaps (CDSs), showing that a high correlation between the reference entity and counterparty can create significant counterparty risks for a buyer of protection. This analysis has been extended to value the counterparty risk tranches of credit portfolios, illustrating that the counterparty risk increases for more senior tranches. Indeed, we have shown that the counterparty risk in so-called super senior tranches is massive.

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