APPENDIX 8A: Formulas for EE, PFE and EPE for a normal distribution

Consider a normal distribution with mean μ (expected future value) and standard deviation (of the future value) σ. Let us calculate analytically the two different exposure metrics discussed. Under the normal distribution assumption, the future value of the portfolio in question (for an arbitrary time horizon) is given by:

$$V = \mu + \sigma Z,$$

where Z is a standard normal variable.

i) Potential future exposure (PFE)

This measure is exactly the same as that used for value-at-risk calculations. The PFE at a given confidence level α, (PFE_α) tells us an exposure that will be exceeded with a probability of no more than $1-\alpha$. For a normal distribution, it is defined by a point a certain number of standard deviations away from the mean: -

$$PFE_\alpha = \mu + \sigma\Phi^{-1}(\alpha),$$

where $\Phi^{-1}(.)$ represents the inverse of a cumulative normal distribution function (this is the function NORMSINV(.) in Microsoft Excel™). For example, with a confidence level of $\alpha = 99\%$, we have $\Phi^{-1}(99\%) = +2.33$ and the worse case exposure is 2.33 standard deviations above the expected future value.

ii) Expected exposure (EE)

Exposure is given by:

$$E = \max(V,0) = \max(\mu + \sigma Z, 0)$$

The EE defines the expected value over the positive future values and is therefore:

$$EE = \int_{-\mu/\sigma}^{\infty} (\mu + \sigma x)\varphi(x)dx = \mu\Phi(\mu/\sigma) + \sigma\varphi(\mu/\sigma),$$

where $\varphi(.)$ represents a normal distribution function and $\Phi(.)$ represents the cumulative normal distribution function. We see that EE depends on both the mean and the standard deviation; as the standard deviation increases so will the EE. In the special case of $\mu = 0$ we have $EE_0 = \sigma\varphi(0) = \sigma/\sqrt{2\pi} \approx 0.40\sigma$.

iii) Expected positive exposure

The above analysis is valid only for a single point in time. Suppose we are looking at the whole profile of exposure defined by $V(t) = \sigma\sqrt{t}Z$ where σ now represents an annual standard deviation (volatility). The EPE, integrating over time and dividing by the time horizon, would be:

$$EPE = \frac{1}{\sqrt{2\pi}}\sigma\int_0^T \sqrt{t}dt/T = \frac{2}{3\sqrt{2\pi}}\sigma T^{1/2} = 0.27\sigma T^{1/2}.$$

APPENDIX 8B: Example exposure calculation for a forward contract.

Suppose we want to calculate the exposure on a forward contract and assume the following model for the evolution of the future value of the contract (V_t): -

$$dV_t = \mu dt + \sigma dW_t,$$

where μ represents a drift and σ is a volatility of the exposure with dW_t representing a standard Brownian motion. Under such assumptions the future value at a given time s in the future will follow a normal distribution with known mean and standard deviation: -

$$V_s \sim N\left(\mu s, \sigma\sqrt{s}\right)$$

We therefore have analytical expression for the PFE and EE following from the formulas in Appendix 8A.

$$PFE_s^{\alpha} = \mu s + \sigma\sqrt{s}\Phi^{-1}(\alpha).$$

$$EE_s = \mu s\Phi\left(\frac{\mu}{\sigma}\sqrt{s}\right) + \sigma\sqrt{s}\varphi\left(\frac{\mu}{\sigma}\sqrt{s}\right).$$

APPENDIX 8C: Example exposure calculation for a swap.

Following the example in Appendix 8B, an approximation to a swap contract is to assume that the future value at a given time s is normally distributed according to:

$$V_s \sim N\left(0, \sigma\sqrt{s}(T-s)\right),$$

where the $(T-s)$ factor corresponds to the approximate duration of the swap of maturity T at time s. This assumes that the expected future value is zero at all future dates which in practice is the case for a flat yield curve. We can show that the maximum exposure is at $s = T / 3$ by differentiating the volatility term:

$$\frac{d}{ds}\left(\sqrt{s}(T-s)\right) = \frac{1}{2\sqrt{s}}(T-s) - \sqrt{s} = 0$$

APPENDIX 8D: Example exposure calculation for a cross-currency swap

Combined the results in the two previous Appendices, we consider a cross currency swap to be a combination of the approximate FX forward and interest rate swap positions.

The FX forward future value follows $V_s \sim N\left(0, \sigma_{FX}\sqrt{s}\right)$ and the interest rate swap follows $V_s \sim N\left(0, \sigma_{IR}\sqrt{s}(T-s)\right)$. Assuming a correlation of ρ between future value of each, the approximate cross-currency swap future value will be given by:

$$V_s \sim N\left(0, \sqrt{\sigma_{FX}^2 s + \sigma_{IR}^2 s(T-s)^2 + 2\rho\sigma_{FX}\sigma_{IR}s(T-s)}\right),$$

which is used to compute the PFE shown in Spreadsheet 8.4.

APPENDIX 8E: Simple netting calculation

We have already shown in Appendix 8A that the EE of a normally distributed random variable is:

$$EE_i = \mu_i \Phi(\mu_i / \sigma_i) + \sigma_i \varphi(\mu_i / \sigma_i).$$

Consider a series of independent normal variables representing transactions within a netting set (NS). They will have a mean and standard deviation given by:

$$\mu_{NS} = \sum_{i=1}^{n} \mu_i \qquad \sigma_{NS}^2 = \sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{\substack{i=1 \\ j>i}}^{n} \rho_{ij} \sigma_i \sigma_j$$

where ρ_{ij} is the correlation between the future values. Assuming normal variables with zero mean and equal standard deviations, $\bar{\sigma}$, we have that the overall mean and standard deviation are given by:

$$\mu_{NS} = 0 \qquad \sigma_{NS}^2 = \left(n + n(n-1)\bar{\rho}\right)\bar{\sigma}^2,$$

where $\bar{\rho}$ is an average correlation value. Hence, since $\varphi(0) = 1/\sqrt{2\pi}$, the overall EE will be:

$$EE_{NS} = \bar{\sigma}\sqrt{n + n(n-1)\bar{\rho}} / \sqrt{2\pi}$$

The sum of the individual EEs gives the result in the case of no netting (NN):

$$EE_{NN} = \bar{\sigma} n / \sqrt{2\pi}$$

Hence the netting benefit will be:

$$EE_{NS} / EE_{NN} = \frac{\sqrt{n + n(n-1)\bar{\rho}}}{n}$$

In the case of perfect positive correlation, $\bar{\rho} = 100\%$, we have:

$$EE_{NS} / EE_{NN} = \frac{\sqrt{n + n(n-1)}}{n} = 100\%$$

The maximum negative correlation is bounded by $\bar{\rho} \geq -1/(n-1)$ and we therefore obtain:

$$EE_{NS} / EE_{NN} = \frac{\sqrt{n - n(n-1)/(n-1)}}{n} = 0\%$$