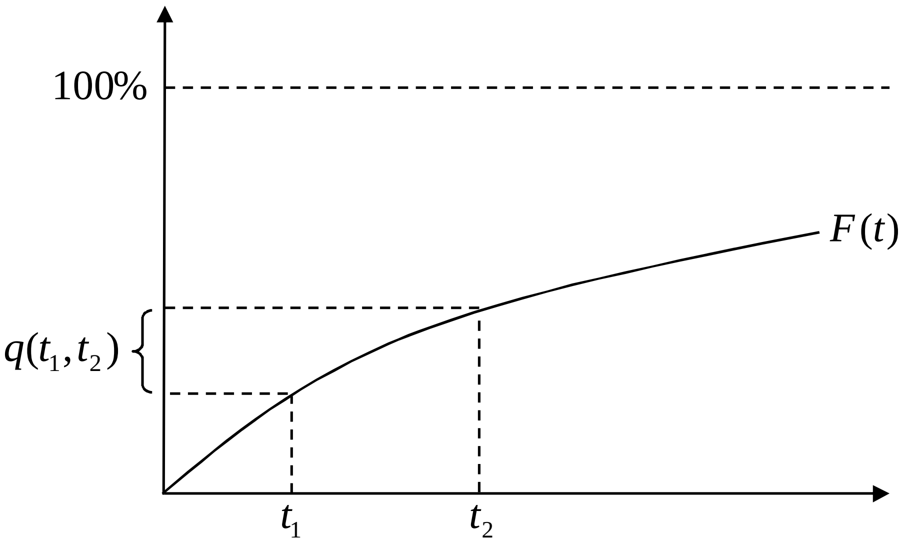
APPENDIX 10A. Definition of cumulative default probability function.

In defining default probabilities, we define a cumulative default probability function, $F(u)$ which (assuming the entity is not currently in default) gives the probability of default at any point prior to time *u*. The marginal default probability, which is the probability of a default between two specified future dates, is given by:

$$q(t_1, t_2) = F(t_2) - F(t_1) \qquad (t_1 \leq t_2)$$

The instantaneous default probability is given by the derivative of $F(u)$.



We also use the definition of survival probability as $S(u) = 1 - F(u)$.

APPENDIX 10B. Mathematics behind the default process and calculation of market-implied default probabilities.

If default is assumed to be a Poisson process driven by a constant intensity of default then the cumulative default probability is:

$$F(u) = 1 - \exp[-hu],$$

where h is the intensity of default, often described as the hazard rate. The instantaneous default probability is:

$$\frac{dF(u)}{du} = h\exp[-hu].$$

Since $\exp[-hu]$ gives the probability of **no** default before date *u*, we can interpret h as a forward instantaneous default probability; the probability of default in an infinitely small period dt conditional on no prior default is hdt.

i) Link from hazard rate to credit spread

We will make the assumption that all cashflows are paid continuously which will simply the exposition. In practice, calculations must account for the precise timing of cashflows (as is done in Spreadsheet 10.2 for example) although the approximation below are reasonably accurate[1].

The risky value of receiving a continuous stream of cashflows can be written as:

$$\int_0^T B(u)S(u)du,$$

where $B(u)$ is the risk-free-discount factor and $S(u) = 1 - F(u)$ is the survival (no default) probability. The above quantity is often called the risky annuity (or risky duration).

The value of receiving protection from a credit default swap (CDS) can be represented as:

$$(1-R)\int_0^T B(u)dF(u) = (1-R)h\int_0^T B(u)S(u)du.$$

The fair CDS spread will be the ratio of the value of default protection divided by the risky annuity (the unit cost of paying for the protection) and we can therefore see that

[1] For example, CDS premium are typically paid quarterly in arrears but an accrued premium is paid in the event of default to compensate the protection seller for the period for which a premium has been paid. Hence, the continuous premium assumption is a good approximation.

$$Spread = (1-R)h \text{ or } h = \frac{Spread}{1-R}$$

ii) Simple formulas

Suppose we define the risk-free discount factors via a constant continuously compounded interest rate $B(u) = \exp[-ru]$. We then have closed-form expressions for quantities such as the risky annuity:

$$\int_t^T \exp[-(r+h)u]du = \frac{1-\exp[-(r+h)u]}{r+h}.$$

iii) Incorporating term structure

For a non-constant hazard rate, the survival probability is given by:

$$S(u) = \exp\left[-\int_t^u h(x)dx\right].$$

To allow for a term structure of credit (for example, CDS premiums at different maturities) and indeed a term structure of interest rates, we must choose some functional form for h. Such an approach is the credit equivalent of yield curve stripping and was first suggested by Li (1998). The single-name CDS market is mainly based around 5-year instruments and other maturities will be rather illiquid. A standard approach is to choose a piecewise constant representation of the hazard rate to coincide with the maturity dates of the individual CDS quotes. This is illustrated in Spreadsheet 10.2.