APPENDIX 11A: Simple formula for the impact of collateral on exposure

Assuming that a netted set of trades is perfectly collateralised at a given time and the change in the netted exposure (and collateral value) follows a normal distribution with zero mean and volatility parameter σ_p, then using the results of Appendix 7A, the potential future exposure at as given confidence level α is given by:

$$PFE_\alpha = \sigma_p \times \sqrt{MP\mathrm{R}} \times \Phi^{-1}(\alpha),$$

where MPR denotes the margin period of risk. The above formula is analogous to a VAR formula under a normal distribution assumption of portfolio value. The EE is given by:

$$EE = \frac{1}{\sqrt{2\pi}} \times \sigma_p \times \sqrt{MPR} \approx 0.4\sigma_E\sqrt{\Delta t}.$$

Given the short period, it is unlikely that the drift of the distribution is likely to be an important consideration.

In the case of a swap, the duration must be considered in order to derive a price volatility. A simple way to do this is to multiply by the remaining maturity giving the approximation shown in Equation (11.4).

$$EE(u) \sim 0.4 \times \sigma_p \times \sqrt{MPR} \times (T - u).$$

APPENDIX 11B: Approximate ratio for uncollateralised and collateralised EPE

It is interesting to assess the reduction of EPE due to a collateral agreement as a function of the margin period of risk (MPR) and maturity of the underlying portfolio. Since CVA is approximately proportional to the EPE then this same reduction can be used to assess the likely impact on CVA. The assumptions in deriving this formula are zero threshold, current MtM of zero and a one-way CSA in our favour (since we do not incorporate the impact of posting collateral ourselves).

i) Uncollateralised case

As discussed in Appendix 7B, a reasonable proxy for the standard deviation of an uncollateralised portfolio is $\sigma\sqrt{t}(T-t)$ where T is the longest maturity in the portfolio and σ is some volatility term (for example for an interest swap portfolio this would be approximately some weighted average interest rate volatility). Under normal distribution assumptions, assuming the current and expected future value of the portfolio is zero then the expected exposure would be $\sigma\sqrt{t}(T-t)/\sqrt{2\pi}$. Integrating this term between now and the final maturity and dividing by the maturity to find the EPE would give:

$$\frac{\sigma\int_0^T \sqrt{t}(T-t)}{T\sqrt{2\pi}} = \frac{4}{15\sqrt{2\pi}}\sigma T^{\frac{3}{2}}.$$

ii) Collateralised case

In the collateralised case, a reasonable proxy for the standard deviation is $\sigma\sqrt{MPR}(T-t)$. Integrating this in a similar manner gives:

$$\frac{\sigma\sqrt{MPR}\int_0^T (T-t)}{T\sqrt{2\pi}} = \frac{1}{2\sqrt{2\pi}}\sigma T\sqrt{MPR}$$

iii) Approximate effect of collateral

Taking the ratio of the above EPE terms would give a factor of:

$$\frac{8}{15}\sqrt{T/MPR} \approx 0.5\sqrt{\frac{T}{MPR}}.$$

Hence, a useful ballpark estimate of the impact of collateral on reduction of EPE (and CVA) would be by a factor $0.5\sqrt{T/MPR}$. The ratio is not surprising since the collateral agreement has the impact of reducing the risk horizon from T to MPR. The factor of 8/15 is due to the uncollateralised profile being assumed to have a classic humped shape (obviously for a portfolio with a monotonically increasing exposure such as one dominated by a long-dated CCS then this factor should be removed).

For example, if the margin period of risk was 20 calendar days and the maturity of the portfolio 5-years then the estimate would give 5.09, i.e. the "collateralised EPE" should be five times smaller than the uncollateralised EPE.

iv) Example

We compare the simple approximation to the actual result for a portfolio of four swaps with a current MTM of zero. A one-way CSA with zero threshold is assumed and a 20-day margin period of risk is chosen. The actual EE profiles without (NC) and with (C) are shown in Figure 11.1A. The actual benefit of collateral (in EPE terms) is calculated as 6.51 and the estimate as mentioned above is 5.09.

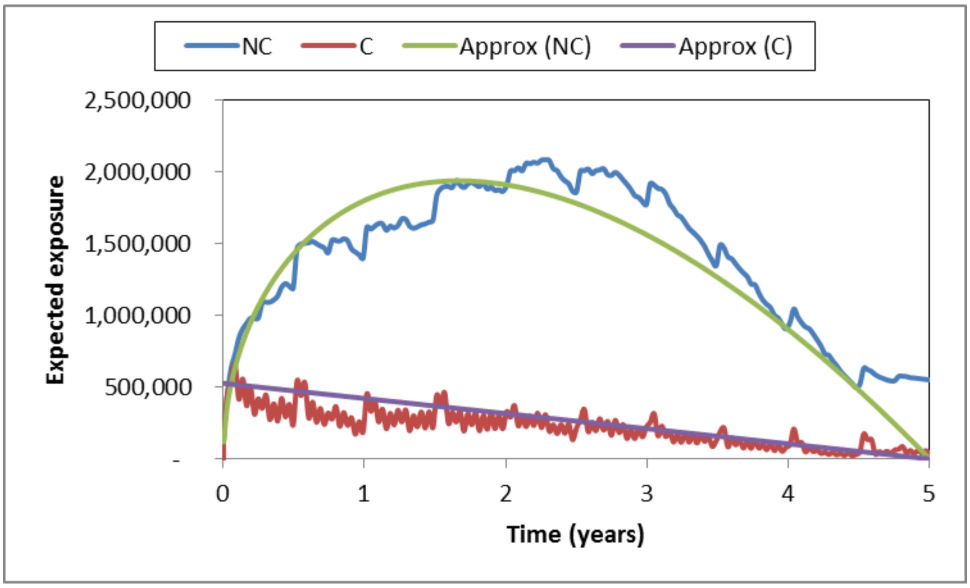

Figure 11.1A. *Illustration of actual and approximate EE profiles for the uncollateralised (NC) and collateralised (C) cases for a real portfolio of four swaps.*

The actual number obvious gets smaller if we introduce a two-way CSA (actual ratio 3.00[1]) and then a bilateral threshold of 2,000,000 (actual ratio 2.30). For portfolios in-the-money then the multiplier will increase due to the immediate receipt of collateral whilst for out-of-the-money portfolios the reverse will happen.

v) Cross currency swap

For a cross-currency swap type profile, we can along similar lines compute a multiplier of:

$$\frac{2}{3}\sqrt{\frac{T}{MPR}}$$

[1] In this portfolio, this is a large effect since the expected future value is negative (ENE greater than EPE) meaning that we are likely to post collateral in many cases.

APPENDIX 11C: Impact of initial margin on exposure

As noted in Appendix 7A, the expected exposure of a normal distribution can be written as:

$$EE = \mu\Phi\left(\frac{\mu}{\sigma}\right) + \sigma\varphi\left(\frac{\mu}{\sigma}\right)$$

For the collateralised case (zero threshold, no initial margin) the impact of the margin period of risk would lead to $\mu = 0$ and $\sigma = \sqrt{\tau_{MPR}}$ giving an expected exposure (EE) of:

$$EE_{no\ IM} = \sqrt{\tau_{MPR}}\varphi(0) = \sqrt{\tau_{MPR}}(2\pi)^{-0.5}$$

The impact of initial margin can be considered equivalent to shifting the mean of the distribution to be $\mu = -\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}$ where τ_{IM} is the time horizon and α the confidence level used to define the initial margin (the initial margin is assumed to be also calculated from normal distribution assumptions potentially using a different time horizon). This leads to an EE of:

$$EE_{IM} = -\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}\Phi\left(\frac{-\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}}{\sqrt{\tau_{MPR}}}\right) + \sqrt{\tau_{MPR}}\varphi\left(\frac{-\Phi^{-1}(\alpha)\sqrt{\tau_{IM}}}{\sqrt{\tau_{MPR}}}\right)$$

This can be simplified to give:

$$EE_{IM} = \sqrt{\tau_{MPR}}\varphi\left(\sqrt{\lambda}K\right) - K\sqrt{\tau_{IM}}\Phi\left(-\sqrt{\lambda}K\right)$$

where $\lambda = \tau_{IM}/\tau_{MPR}$ is the ratio of the time horizon used (τ_{IM}) for the IM calculation divided by the MPR for the exposure quantification (τ_{MPR}) and $K = \Phi^{-1}(\alpha)$ where $\varphi(.)$ is a standard normal density function and $\Phi(.)$ is the cumulative standard normal density function.

Finally:

$$R_\alpha = \frac{EE_{no\ IM}}{EE_{IM}} = \left[\varphi\left(\sqrt{\lambda}K\right) - K\sqrt{\lambda}\Phi\left(-\sqrt{\lambda}K\right)\right]^{-1}(2\pi)^{-0.5}.$$