A Trick of the Credit Tail

Published in Risk Magazine, March 2008

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Leveraged super senior (LSS) trades represent a mechanism for packaging senior credit risk. A significant volume of LSS structures have been issued to date and yet there seems to be no formal pricing approach. In this article we discuss the valuation of LSS protection in a model independent framework. We argue that the "equivalence" approach to pricing that seems widely used is not appropriate.

The structured credit market has grown rapidly in recent years with the synthetic CDO product which allows issuers to sell a particular tranche of a portfolio which they hedge with more simple instruments such as single name CDS. One problem in the early development of the CDO market was the fact that correlation was a key input to the pricing but was a rather opaque quantity. The development of the index tranche market in 2004 provided a solution to this problem of observability and has led to correlation trading across the capital structure for corporate credit portfolios and other asset classes such as ABS, leveraged loans and CMBS.

One feature of the correlation market is that senior risk trades at a significant risk premium. Take the iTraxx [22-100%] tranche as an example: this 125 investment grade name portfolio will require 40 credit events at 30% average recovery value before suffering any loss[2]. Almost one third of the portfolio needs to default before this tranche loses principal and the likelihood of this might be considered negligible by market participants when taken into consideration alongside factors such as their own financial solvency.

Even if an investor holds the view that a super senior tranche has no chance of ever suffering a loss, there is still the problem of mark-to-market price volatility, cost of capital and other aspects. These points tend to keep super senior spreads relatively wide as characterised by the high implied correlation in the senior region. For these reasons, there is incentive to develop other ways to package senior risk. One such example is the leveraged super senior (LSS) transaction. In this article, we will present a quantitative analysis on the pricing of protection purchased via LSS structures, noting that some the conclusions have broader implications, for example

[1] I am grateful to Michael Walker for comments on a preliminary draft of this paper and providing some pricing results. I also thank participants at the WBS Fixed Income conference 2007, the ICBI Global Derivatives and Risk Management conference 2007 and the ICBI Ri$kminds conference 2007 for interesting discussions on this topic. Detailed initial discussions with David Shelton and comments from two anonymous referees are gratefully acknowledged.

[2] $22\% \times 125 / (1 - 30\%) = 39.3$

for structurally similar structured investment vehicles (SIVs) and credit derivative product companies (CDPCs). We will argue that the standard approach of pricing a LSS as being equal to the equivalent un-leveraged value less some "gap risk" is inherently inconsistent and could lead to some unpleasant surprises for LSS issuers[3].

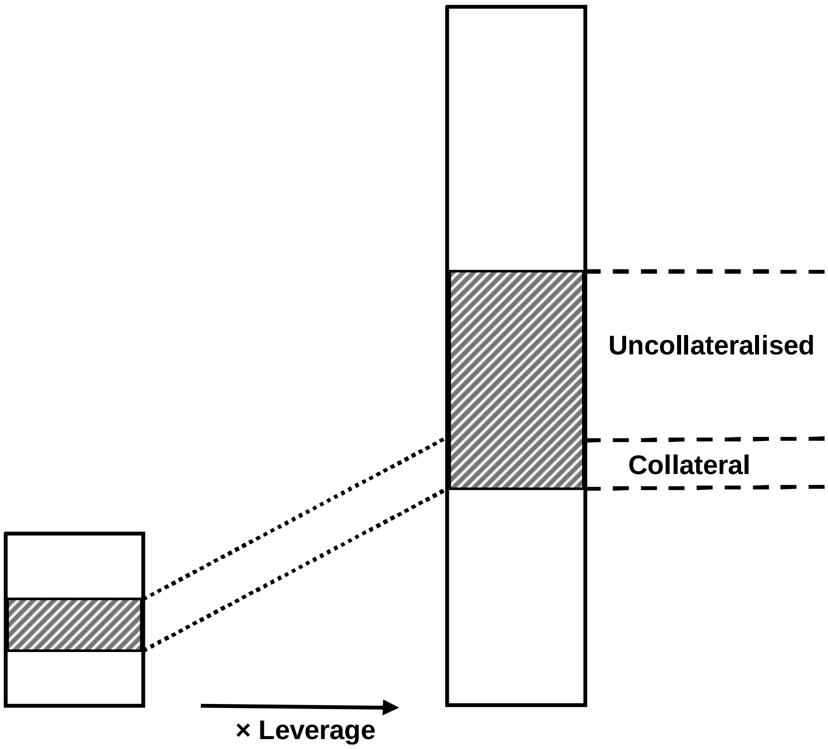
This article is written at a time where LSS have suffered problems arising from the turbulence of July and August 2007 which created significant mark-to-market losses from a position taking super senior credit risk (a result of spread widening and increases in implied correlation). Our focus will be a robust theoretical pricing study and not other qualitative aspects such as rating agencies approaches and problems arising from the disruption in the Canadian conduit market.

The Leveraged Super Senior Structure

The premise of LSS is that super senior spreads in un-leveraged form do not have the correct risk-return profile to many investors since their premium is too small and the issuer therefore applies leverage to the product to create a more attractive return. The leverage in a LSS transaction reflects the fact that the investor's cash participation is less than the notional of the super senior tranche. For example a \$10 million investment may be leveraged 10 times into a super senior tranche with a notional of \$100 million. The investor has sold protection on \$100 of protection but posted only \$10 initial collateral. Generally, for a leverage of x times, the investor will initially commit $1/x$ units of collateral as illustrated in Figure 1. LSS trades have mostly been structured on corporate credit but also more recently on ABS portfolios.

[3] In this article for LSS we will typically use "issuer" to refer to the protection buyer and "investor" the protection seller.

Figure 1. Illustration of the LSS concept. A tranche is effectively multiplied by a leverage factor and the resulting tranche is collateralised as shown.



There needs to be a mechanism to mitigate the risk that the LSS issuer retains via the uncollateralised exposure. This is achieved using a "trigger event" where the investor might have the option to de-leverage by posting more collateral but will otherwise face the structure being unwound by the issuer at prevailing market rates.

To understand the LSS trigger mechanisms, note that the value of a super senior tranche depends on portfolio losses, average portfolio spread level[4] and base correlation (more generally dependency between credit events) as well as remaining maturity. In defining a trigger, the issuer is trying to ensure that the unit value of the tranche will always be below $1/x$ with the likely incorporation of some cushion that may be appropriate given the risk in unwinding the trade. The trigger definition represents a balance between a simple definition that may ease documentation, understanding of the product and the ratings process and a more complex one that leaves less unwind risk for the issuer. It is useful to have the possible trigger events in mind and so we briefly describe the typical mechanisms used in the market.

Loss only trigger. In this case the trigger is defined by a certain loss on the portfolio (which may increase over time to reflect time decay). The rating process for such a structure is rather easy since the payoff is similar to that of a CDO. However, the issuer is potentially heavily exposed from movements in the underlying spreads and implied correlation levels.

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[4] Making the assumption that all spreads are equal to the average spread is conservative for the following purpose as this maximises the value of a senior tranche.

Spread triggers. Here, the trigger is defined by an average portfolio spread as a function of portfolio loss and time to maturity. Although there is less uncertainty, the issuer still has risk over the correlation at the trigger time. The rating model for such a structure is significantly more complex since it requires the portfolio spread process (and its relationship to default losses) to be modelled.

Market value trigger. Finally, some structures reference the market value (PV) of the tranche directly. This guarantees the cushion available when the trigger is hit although some "gap risk" still exists for the issuer. The ratings agencies have at the time of writing not typically rated such structures[5] perhaps due to a reluctance to model implied correlation levels.

Pricing CDO Tranches

More details on CDO tranche valuation can be found, for example, in Laurent and Gregory [2005]. Suppose the underlying tranche is defined by losses in $[A,B]$. The tranche loss process $M_{A,B}(t)$ is given by: -

$$M_{A,B}(t) = \min\left((L(t)-A)_+, B-A\right) \equiv \min(L(t),B) - \min(L(t),A), \tag{1}$$

where $L(t)$ is the cumulative portfolio loss at time t and $y_+ = \max(y,0)$. The value at time t of the loss leg of such a tranche with maturity T is then essentially a integration over the loss distribution $L(t)$ of the portfolio in question: -

$$V_{A,B}(t) = E^Q\left[\int_t^T B(t,s)dM_{A,B}(s)\right], \tag{2}$$

where $B(t,s)$ represents the risk-free discount factor. In order to compute $V_{A,B}(t)$ and more complex quantities, we need a model to represent $L(t)$. For the purposes of this paper we will restrict ourselves to a model independent analysis.

The "Equivalence" Pricing Approach

Since we denote the leverage of the structure as x, the initial investment (or collateral) will be $(B-A)/x = \alpha$. We denote the time of unwind as τ and use τ^+ to indicate that there will be some unwind period[6]. We make no specific assumptions on

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[5] With the exception of DBRS the Canadian rating agency that rated LSS transactions for market value triggers for a time and then stopped in January 2007.

[6] Strictly speaking there are two components. Except in a market value trigger, the trigger may be badly specified so that the issuer is already losing money at the trigger time, i.e. $V_{A,B}(\tau) > \alpha$ or it may be only gap risk arising from the fact that $V_{A,B}(\tau) \le \alpha < V_{A,B}(\tau^+)$. Often both components will be referred to generically as "gap risk". The term $V_{A,B}(\tau^+)$ contains all costs as a result of unwinding the structure.

the trigger type and thus the analysis covers all three triggers previously defined (and any others that may exist). We focus solely on the value of the protection leg of each tranche since this is the key component in the analysis. Where relevant we will comment on the impact of the premium leg component in the pricing relationships derived. We denote by $\tilde{V}_{A,B,\alpha}(t)$ the time *t* value of leveraged protection for leverage defined by α.

In defining the trigger, the issuer will aim to ensure that $V_{A,B}(\tau^+) < \alpha$ in order to fully mitigate their risk. The issuer's position often seems to be argued as being long protection on the full notional (the equivalent un-leveraged protection) less a "gap option" with strike α referenced to the underlying tranche value $V_{A,B}(\tau^+)$: -

$$\tilde{V}_{A,B,\alpha}(t) \equiv V_{A,B}(t) - \underbrace{E^P[1_{\tau<T} B(t,\tau^+)(V_{A,B}(\tau^+) - \alpha)_+]}_{\text{"gap option"}}, \tag{3}$$

where $1_{\tau<T}$ is an indicator function that takes the value 1 if the trigger event occurs before maturity. The gap option might be priced under the physical measure as illustrated. The issuer is short the gap option due to the limited recourse feature of the trade which means that the investor is not responsible for any losses when $V_{A,B}(\tau^+) > \alpha$. In this framework, the issuer might argue that the short gap option has minimal value. This would presumably be the case if the trigger is well-defined so that there is ample cushion available. On the other hand, the issuer can also minimise the value of this option by ensuring that the *probability* of hitting the trigger is small. This seems to contradict the intuition that it is better to unwind such a structure sooner rather than later.

In the following, we will describe a model and trigger independent analysis of LSS pricing for which there is one key assumption related to the trigger event. In some structures, unwind will be automatic whereas in others the investor will have the option to de-leverage via posting more collateral at pre-specified levels. We argue that whether or not the investor has this choice, to de-leverage is sub-optimal[7] compared to unwinding and investing in a new LSS. This point may be debated and we should comment that in the recent market volatility, many LSS may have hit their triggers and investors may have chosen to de-leverage rather than face unwind in a market where there was virtually no liquidity on senior tranches. Such sub-optimal effects are not uncommon in other areas such as the convertible bond, Bermudan swaption and equity index option markets. However, in a rigorous pricing framework we must assume optimal decisions and frictionless markets. Put another way, issuers cannot mark-to-market their leveraged super senior protection based on the assumption of sub-optimal behaviour by the investor.

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[7] Even if the investor wants to remain a taker of super senior risk, their optimal strategy is to unwind and execute another LSS at a more attractive premium (and return on investment). When the investor unwinds then their losses are capped at α whereas if they de-leverage then they increase their potential losses without receiving any additional return (furthermore, the investor will typically not get the full gain from de-leverage i.e. they will be required to inject more collateral than the increase in trigger level).

Formal Pricing Approach

a) Loss only triggers

Let us start from the simplest loss-only trigger with the trigger level denoted by K (the case of a time-dependent trigger is not substantially more complex). In this case, by construction the tranche cannot experience losses before the trigger is breached as long as $K < A$, i.e. the trigger is less that the attachment point of the tranche. This means that the protection buyer does not have any component corresponding to tranche losses similar to equation (2) but instead an option to exercise and receive the market value of the tranche at the trigger time. If the trigger is hit then the protection buyer will receive the value of the protection at time τ^+ (i.e. including unwind costs), up to the value of the collateral α. The value of the LSS protection is: -

$$\tilde{V}^{LT}_{A,B,\alpha}(t) = E^Q\left[1_{\tau<T}\, B(t,\tau^+) \min\left(V_{A,B}(\tau^+),\alpha\right)\right]. \quad (4)$$

Using the fact that $\min(a,b) = a-(a-b)_+$, we can show the origins of the gap option as characterised by equation (3).

$$\tilde{V}^{LT}_{A,B,\alpha}(t) = E^Q\left[1_{\tau<T}\, B(t,\tau^+) V_{A,B}(\tau^+)\right] - E^Q\left[1_{\tau<T}\, B(t,\tau^+)\left(V_{A,B}(\tau^+)-\alpha\right)_+\right]. \quad (5)$$

The payoff $E^Q\left[1_{\tau<T}\, B(t,\tau^+)\alpha\right]$ provides a superhedge for the value of the loss trigger LSS protection. This corresponds to a contract that pays to the issuer the amount of initial collateral α at the trigger time and can be priced[8] as a digital CDO tranche with attachment point equal to the loss trigger and a notional of α. We can represent by $V_{K,K+\varepsilon}(t)/\varepsilon$ the unit value of digital CDO tranche for small ε. Hence we have the following inequality[9]: -

$$\tilde{V}^{LT}_{A,B,\alpha}(t) \le (\alpha/\varepsilon) V_{K,K+\varepsilon}(t). \quad (6)$$

Since leveraged protection can never be worth more than the equivalent unleveraged protection (this might be reasonably obvious but for the avoidance of doubt is shown in the general case later) we can identify the point at which $V_{A,B}(t) = (\alpha/\varepsilon)V_{K,K+\varepsilon}(t)$ as being significant since it effectively defines a maximum leverage[10] for the transaction as the leveraged protection must be worth no more than the superhedge.

[8] With the usual problems associated with tranchelet pricing such as the interpolation of base correlation points. As mentioned earlier, the loss trigger levels may change over time in which case we need to price the more complex variable subordination tranche.

[9] To convert this relationship to one involving tranche spreads we must account for the fact that the premium legs can match so we should solve for the spread of a tranche paying a premium indexed to losses in the range $[A,B]$ but with the digital protection leg that pays α as soon as the loss trigger is hit.

[10] By this we mean that there is a maximum leverage above which the issuer cannot argue under any circumstances that $\tilde{V}^{LT}_{A,B,\alpha}(t) = V_{A,B}(t)$.

Using the previous definition of α we then obtain the following expression for the maximum leverage of a LSS based on tranche $[A,B]$ with loss trigger K : -

$$x^{*}_{A,B,\alpha,K}(t) = \frac{V_{K,K+\varepsilon}(t)}{V_{A,B}(t)} \frac{(B-A)}{\varepsilon}. \tag{7}$$

Any leverage in excess of this quantity would violate the arbitrage condition imposed via (6) and enable the LSS protection seller to super-hedge with a digital tranche (and construct an arbitrage).

In the case of a loss-only trigger LSS, we can apply the equivalence pricing approach represented by equation (3) but we must price under the risk-neutral measure. In this case the presence of a cheaper super-hedge would be seen via a significant (negative) contribution from the gap option. Pricing the gap option under the physical measure is not appropriate since even if the maximum leverage is not breached at trade inception, it may be later on as we will illustrate in a later example. The right way to price a loss-trigger LSS is therefore as a tranche option using an appropriate model for the dynamics of $L(t)$. We note that the hedging of this payoff will be a challenge and unpleasant gamma effects might be seen due to the super-hedge.

Our model-independent arguments can be linked to recent dynamic modelling approaches such as Hull and White [2007] and Walker [2007] on loss-trigger LSS. Indeed the superhedge can be seen in the results of Walker [2007] since at sufficiently high leverage and/or loss trigger level the protection value varies exactly as predicted by equation (6) which is linear in α and therefore inversely proportional to the leverage x.

b) More complex triggers

In the case of triggers that are not purely loss based (spread and market value), we must make a more general analysis to account for the fact that tranche losses may occur before the trigger is hit. This can be thought of as being equivalent to the standard CDO protection as priced in equation (2) for the collateralised losses in $[A, A+\alpha]$ conditional on the trigger event having not previously occurred. We represent the value of the LSS protection in this more general case as the sum of the following two components, the first corresponding to scenarios before the trigger is hit and the second to the trigger scenario as before : -

$$\tilde{V}_{A,B,\alpha}(t) = E^{Q}\left[1_{\tau>s}\int_{t}^{T} B(t,s)\,dM_{A,A+\alpha}(s)\right] + E^{Q}\left[1_{\tau<T} B(t,\tau^{+})\min\left(V_{A,B}(\tau^{+}),\alpha\right)\right] \tag{8}$$

Compared to the equivalence approach, the above pricing formula represents a very different perspective on the LSS valuation in that it is equal to the collateralised protection and the option to receive the minimum of the market value of the tranche and the collateral α at the trigger time. This could be the view taken by the investor where the "trigger option" represented by the last term in equation (8) is argued to have minimal value. Indeed this is exactly the component assessed by rating agencies for typical LSS structures which can achieve triple-A ratings by virtue of quantification of the above equation (for example see Chandler et al. [2005]) under the physical measure[11]. Negotiations around the problems in the Canadian conduit market have involved the replacement of mark-to-market triggers by "more remote spread loss triggers". If this is the case then issuers should be taking losses on LSS structures due to giving up option time value according to the second term in equation (8). At the time of writing, the triggers have even been temporarily removed which is not a choice supported by a rigorous pricing approach.

Assuming we use the equivalence approach under the risk-neutral measure. Taking the difference between this pricing in equation (3) and equation (8): -

$$
\begin{aligned}
&E^Q\left[\int_t^T B(t,s)dM_{A,B}(s)\right] - E^Q[1_{\tau<T}B(t,\tau^+)(V_{A,B}(\tau^+)-\alpha)_+] \\
&-E^Q\left[1_{\tau>s}\int_t^T B(t,s)dM_{A,A+\alpha}(s)\right] - E^Q\left[1_{\tau<T}B(t,\tau^+)\min\left(V_{A,B}(\tau^+),\alpha\right)\right] \\
&=\left(E^Q\left[\int_t^T B(t,s)dM_{A+\alpha,B}(s)\right] - E^Q[1_{\tau<T}B(t,\tau^+)V_{A+\alpha,B}(\tau^+)]\right) \\
&+\left(E^Q\left[\int_t^T 1_{\tau<s}B(t,s)dM_{A,A+\alpha}(s)\right] - E^Q[1_{\tau<T}B(t,\tau^+)V_{A,A+\alpha}(\tau^+)]\right)
\end{aligned}
\tag{9}
$$

The equivalence pricing approach claims that the issuer has a full claim on the losses in the range $[A+\alpha, B]$ through the term $E^Q\left[\int_t^T B(t,s)dM_{A+\alpha,B}(s)\right]$ whereas in reality a claim can only be made on the present value of the protection corresponding to these losses at the trigger time via $E^Q[1_{\tau<T}B(t,\tau^+)V_{A+\alpha,B}(\tau^+)]$. This shows that the equivalence approach is definitely not valid for pricing LSS contracts on structures using the more complex spread and market value triggers.

Pricing Bounds

From equation (8), we have the (perhaps obvious) bounds for the value of the leveraged protection[12].

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[11] Since rating agencies focus solely on expected loss or probability of loss and not mark to market volatility, the use of the physical measure is appropriate. Rating agencies seem to (conservatively) assume zero recovery.

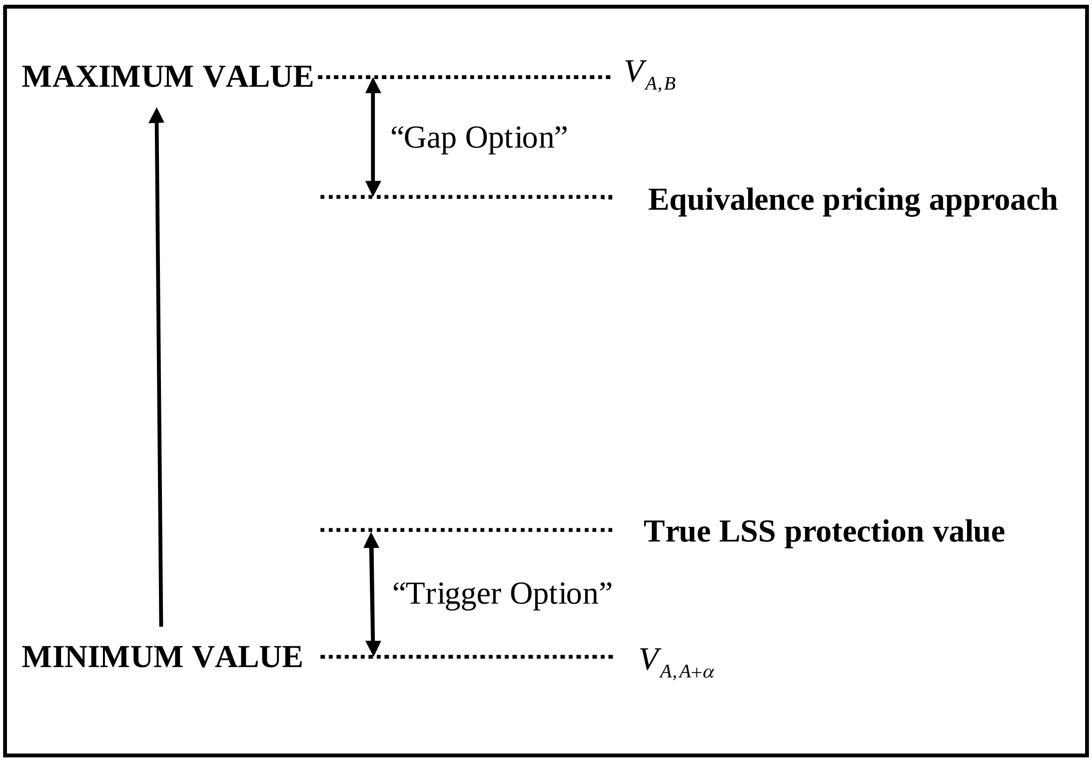
[12] In relating the value of protection to tranche spreads we must be careful that both the upper and lower bounds correspond to contracts with a premium leg referenced to the full notional of $(B-A)$. This means that the upper bound spread would be exactly the fair spread on the un-leveraged $[A,B]$ tranche whereas the lower bound spread would correspond to a tranche with premium leg referenced to a $[A,B]$ tranche but protection leg referenced to $[A,A+\alpha]$ losses. We must also condition on no trigger event in the premium leg calculation.

$$V_{A,A+\alpha}(t) \leq \widetilde{V}_{A,B,\alpha}(t) \leq V_{A,B}(t), \qquad (10)$$

In the case of no trigger $(E[\tau] \to \infty)$, the investor will suffer losses on the tranche as they occur but only up to the amount of the initial investment α. At the other extreme, the LSS protection can be worth no more than the equivalent un-leveraged protection which can be seen from the case where the trigger happens instantaneously $(E[\tau] = t)$ with $V_{A,B}(t) < \alpha$ (to structure the deal otherwise would be rather naïve). We note that this upper bound is **not** obtained by the assumption of $E[V_{A,B}(\tau^{+}) > \alpha] = 0$ as in the equivalence pricing argument even though this is the situation of zero gap risk.

An overview of the LSS pricing is illustrated in Figure 2 for the most popular spread and market value trigger types. We see that the equivalence approach represented by equation (3) is incompatible with the more formal treatment. Taken to an extreme, with no trigger, we have $E(1_{\tau<T}) = 0$ and both gap and trigger options are worthless. If there is little chance of hitting the trigger, the equivalence pricing approach suggests that there is only a small amount of gap risk and the LSS protection is close in value to the equivalent un-leveraged protection. Quite the opposite is true; in order to argue that leveraged protection is close to the value of $V_{A,B}(t)$ then the (risk-neutral) probability of hitting the trigger must be significant.

Figure 2. Schematic illustration of leveraged super senior valuation for spread and market value triggers.



The difference in valuation represented by Figure 2 comes about from the fact that the equivalence pricing argument implicitly assumes that, rather than unwinding the structure, the investor will choose to post more collateral and indeed that they will continue to do this up to the full value of the leveraged exposure (so if they are ten times leveraged they will voluntarily post up to nine times collateral for every initial unit of investment).

We finally note that this analysis has implicitly assumed a continuity of $V_{A,B}(t)$, most significantly ignoring the chance of a large jump in losses through the trigger. For example, a systemic shock involving multiple defaults or Armageddon scenario under risk-neutral dynamics would be extremely unpleasant for the LSS issuer as the losses may hit the uncollateralised portion of the tranche with no chance of mitigating action. Such points can be linked to the default contagion necessary to fit the index tranche market as, for example, described by Laurent et al. [2007]. Such contagion implies that the dynamics at the trigger time dictate that the issuer's expected losses on unwinding are high even if the amount of cushion as seen from today's perspective looks conservative.

Example

We now test the theoretical ideas through a real example using a loss only trigger structure which is the simplest case with which to illustrate the key points. We choose market data for the iTraxx tranche market for three separate dates as given in Table 1. The dates correspond to before the so-called "correlation crisis" of May 2005, before the 2007 "subprime crisis" and finally a more recent data set.

Table 1. Market prices corresponding to the 5-year iTraxx index tranches. All prices are in basis points per annum (bp pa) except the equity tranches which are upfront assuming a 500 bp pa premium according to market convention. The [22-100%] premiums, typically not observed in the market, have been implied by reference to the index level shown.

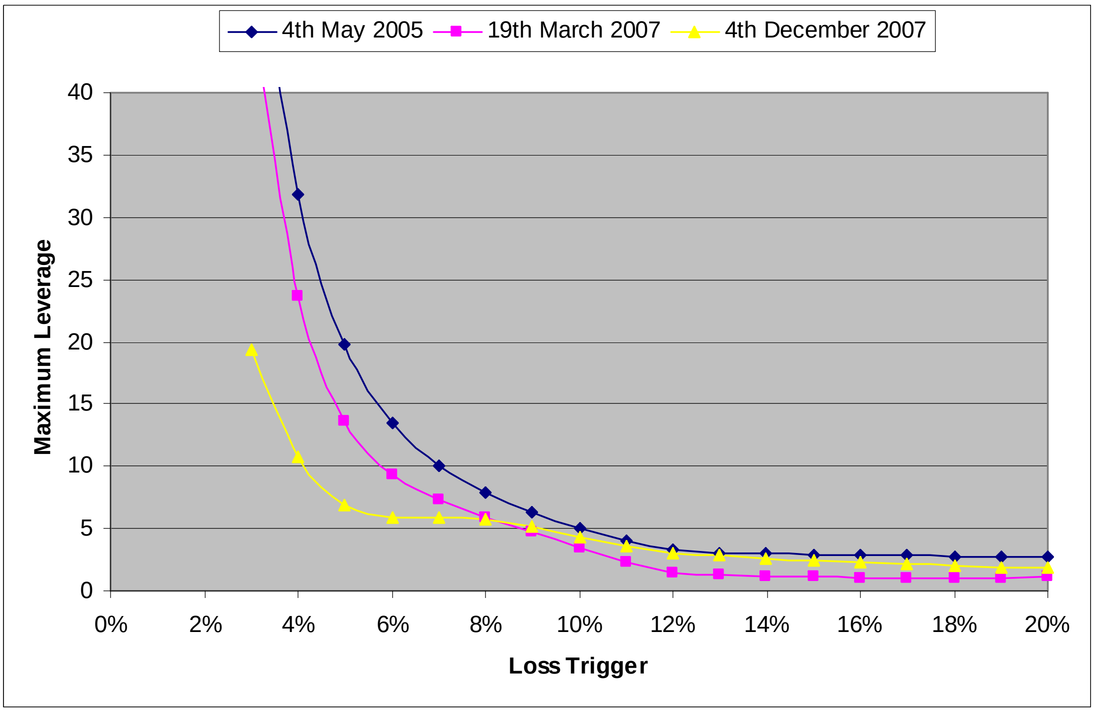
	4th May 2005	19th March 2007	4th December 2007
Index	44	25	53
[0-3%]	29.00%	11.87%	25.05%
[3-6%]	168	54.5	156
[6-9%]	49	14.8	85
[9-12%]	25	6.8	60
[12-22%]	16	2.8	34
[22-100%]	5.5	2.4	15

Consider a 5-year maturity LSS tranche based on the [22-100%] tranche with a loss-only trigger. We calculate[13] the maximum leverage at inception as a function of the loss trigger as defined by equation (7). These results are illustrated in

Figure 3. Not surprisingly, when the loss trigger is at low levels then the maximum leverage is high, reflecting the fact that the super-hedge is very costly. However, the level drops significantly as the loss trigger is moved upwards. For a constant loss trigger of 10% (corresponding to the assumption that the trigger is hit with just over half the original subordination remaining) a leverage of 5 times is not even possible. We can also notice that a steepening of the correlation curve generally causes the maximum leverage to drop which we could associate with the fact that the market implied prices are more "systemic". For example, consider a loss trigger at 5%, which is less than a quarter of the entire subordination. This would permit a ten times leverage until the most recent date when the maximum leverage has dropped to around six times.

[13] We did this using the fairly standard base correlation and Gaussian copula model approach calibrated to the market prices with the correlation interpolated using a cublic spline. An example spreadsheet is available from the author on request.

Figure 3. Illustration of the maximum leverage possible for a loss trigger LSS based on a [22-100%] tranche as a function of the loss trigger level.



Conclusion

A clear conclusion of this analysis is that no LSS structure can be treated by an issuer as a purely "gap risk" product with the gap risk priced under the physical measure. In a loss-only trigger, the risk-neutral measure must be used for pricing the gap option, which may turn out to have significant value due to the superhedge of this payoff. For the more complex trigger types, which have been more common, there are additional problems due to the implicit assumption that the protection seller will always choose to de-leverage the structure. So for spread and market value triggers, the equivalence pricing approach is wrong and the correct approach is rather different as illustrated by Figure 2. There has been a clear lack of solid theoretical foundations in pricing LSS products (and the related structures such as SIVs and CDPCs) which has certainly not helped the recent credit problems. As we showed in a practical example, pricing LSS is not purely related to subjective assessments on gap risk, unwind periods and liquidity. Given the ongoing advances in portfolio credit modelling we may be hopeful of a lot more modelling effort going into pricing the approximately estimated $100 billion of LSS protection that exists in the market today and applying more robust quantitative approaches to the SIV structures, CDPC vehicles and other related structures that are common in the credit world.

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