A Free Lunch and the Credit Crunch

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Monoline insurers act as triple-A guarantors of the rather senior risks in structured finance. The purchaser of credit insurance or protection from a monoline may argue that they take only a small amount of counterparty risk which is a common side-effect of trading OTC derivatives products. However, in this article we argue that credit insurance purchased in this fashion carries significant counterparty risk and from a quantitative point of view has little or no value.

The structured credit market has grown rapidly in recent years and correlation is now traded across the capital structure, from risky equity tranches, to senior tranches that are triple-A rated[1]. Consider the [30-100%] tranche of the North American investment grade portfolio (CDX IG). An investor taking the risk on this tranche can potentially survive default of half the underlying portfolio before losing any principal[2] and may therefore be viewed as a "risk-free" investment. Judged on this basis, the relatively small but still significant premiums paid by such senior tranches could be thought of as a "free lunch". However, there are still problems such as cost of capital and price volatility. A seller of super senior protection will make significant mark-to-market losses when spreads widen (though this is probably not tied to a belief that the tranche is likely to suffer losses). An ideal trade would be to sell super senior protection but not have to recognise mark-to-market changes since the tranche will eventually "pull-to-par". Since senior tranches pay relatively low premiums, a significant leverage will be required (to turn a few crumbs into lunch).

In a previous article (Gregory [2008]), we have presented a quantitative analysis of the pricing of protection purchased via so-called leveraged super senior (LSS) structures and shown that the problem is more complex than might at first be thought. Rather than a LSS being worth the equivalent amount of standard protection minus some "gap risk", it is worth a much smaller value corresponding to the collateralised protection plus a complex "trigger option" arising from the protection buyer's right to unwind the structure via some pre-determined mechanism. In this article we will argue that obtaining senior credit protection from a CDPC or monoline can essentially be thought of as executing a more complex and opaque LSS structure. We then argue that the assumption that such protection can be priced via simply assuming a (small) counterparty risk adjustment is incorrect.

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[1] We may also use the phrases super senior and super triple-A but these are not particularly well-defined so for brevity we use senior and triple-A to cover a broad spectrum of well-collateralised tranche products.

[2] Assuming an average recovery value of 40%.

CDPCs and Monolines

Monoline insurers are financial guarantee companies that are triple-A rated and provide insurance for investment grade transactions in structured finance such as asset-backed securities (ABS) and collateralised debt obligations (CDOs). Credit derivative product companies (CDPCs) are similar in concept but take on risk in the form of derivative contracts rather than insurance policies. A CDPC is effectively a special purpose entity set up to invest in credit derivatives products on a leveraged basis. CDPCs sponsors include asset managers, hedge funds, insurers and banks. Both monolines and CDPCs can be regarded as having a business model that aims to take advantage of the relatively high risk premium component embedded in senior credit protection. CDPCs and monolines will potentially benefit most from assets offering the most substantial risk premiums and therefore may find senior tranches the most attractive investments. They will typically sell protection on a variety of underlyings such as corporate, sovereign and asset-backed securities in single-name or portfolio form.

A monoline or CDPC will typically achieve a triple-A rating based on adhering to strict operating guidelines and a capital adequacy model agreed with the relevant rating agencies (rating agencies consider other aspects such as funding, legal risks, operating guidelines, management structure and regulation). The guidelines aim to discourage the required capital exceeding the actual equity capital. Most importantly, CDPCs and monolines will not post collateral[3]; this point both allows and probably also requires them to achieve a triple-A rating. The capital held by a monoline or CDPC will be small in comparison to the notional of contracts on which they sell protection, indicating a high leverage for the company.

We will restrict ourselves to a theoretical analysis which effectively considers only the quantifiable risks for a counterparty buying protection from a CDPC or monoline. There are many CDPCs and monolines with contrasting business models and operating environments. We focus on the role of monolines and CDPCs as takers of the rather systemic senior credit risk and will base the analysis on the generic structure outlined below describing the operation of a "credit insurer".

Normal state. The credit insurer will be typically triple-A rated by virtue of a ratings-based capital model run daily for the exposures it faces. As long as the required capital does not exceed the actual capital held, the company can operate within its normal operating guidelines.

Restricted state. Typically this is invoked if a capital breach has occurred and will result in restrictions on investments and funding. There will be a certain period allowed (which can be as short as a few days or as long as several months and may be subject to rating agency discretion) for the company to return to normal operating mode. To do this would presumably require raising additional capital or restructuring / unwinding existing trades.

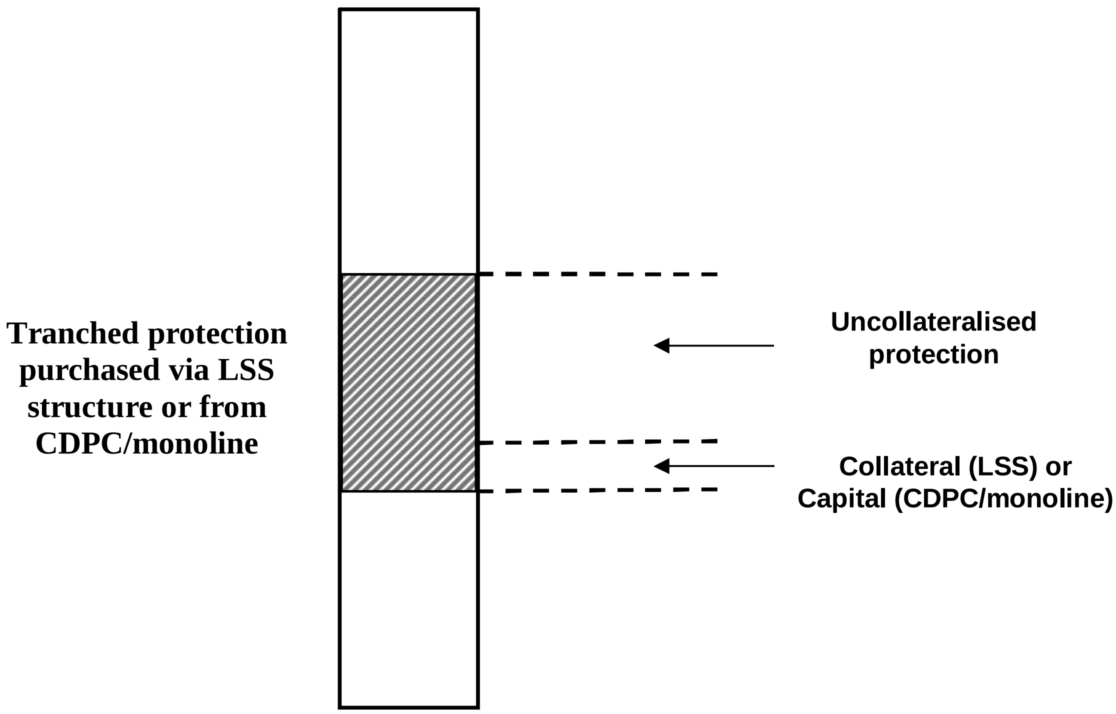
[3] This is the case when they are in normal operating mode. Events such as ratings downgrades may require collateral to be posted.

Run-off / wind-down state. This corresponds to a hibernation state where the credit insurer will be essentially static and trades will gradually mature and any default losses will be settled as and when they occur (assuming there is equity capital to cover them). In some cases there may be a termination mode where they will be required to liquidate all trades.

Comparison to LSS

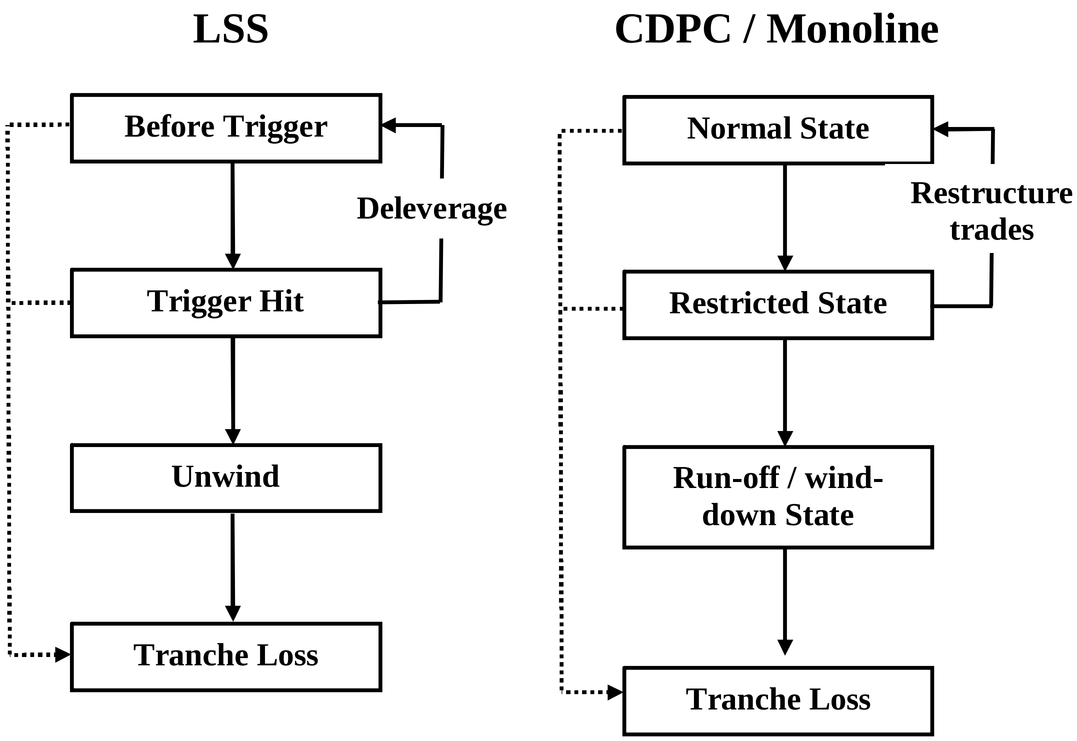
As we show in Figure 1, the LSS structure is a simpler version of a credit insurer. A LSS is essentially a product where an investor sells super senior credit protection in a leveraged format. For an initial leverage of x, the initial collateral will be $1/x$ for each unit of notional. Consider a credit insurer which trades just a single tranche; its equity capital is effectively the same as the collateral in a LSS. In a LSS transaction there are a variety of trigger mechanisms that allow the protection buyer to unwind the transaction to mitigate the uncollateralised exposure. We can therefore see the operating modes above as mimicking this trigger mechanism, i.e. encouraging mitigating action before losses hit the tranche.

Figure 1. Illustration of LSS and simplified CDPC or monoline (in the general case they sell protection on many different tranches).



We illustrate the operational comparison schematically in Figure 2. In a LSS, there is a trigger at which the investor may have the option to de-leverage by injecting more capital into the trade and otherwise will have the trade unwound by the issuer. The restricted state is analogous to the trigger of the LSS. In the same way that a LSS may de-leverage, a credit insurer can recover from the restricted state which may also require a de-leveraging as a result of unwinding or restructuring the trade[4] or by raising additional capital. Finally, just as a LSS may unwind fully, so a credit insurer can reach the termination or wind-down state.

Figure 2. Comparison of LSS and CDPC / monoline operational modes.



Derivatives Counterparty Risk

The pricing of derivatives counterparty risk is a well-studied topic. The risky price of such contracts $\tilde{V}(t)$ is often expressed in the following way: -

$$\tilde{V}(t) = V(t) - EL(t) \qquad EL(t) = (1-\delta)E^{Q}[1_{\tau<T}\, B(t,\tau)[V(\tau)]_{+}], \tag{1}$$

where $V(t)$ is the risk-free value of a single (or group of netted) derivative contract(s) with (maximum) maturity T, $EL(t)$ is the expected loss of the contract(s) over the whole lifetime, δ is a recovery fraction, τ is the counterparty default time and $B(t,\tau)$ is the risk-free discount factor. The expected loss component can be rather complex as it should account for the variability (and negativity) of $V(t)$, the credit quality of the counterparty, collateralisation, and the correlation between the two. Due to netting and collateralisation, the value of $EL(t)$ is often considered to be relatively

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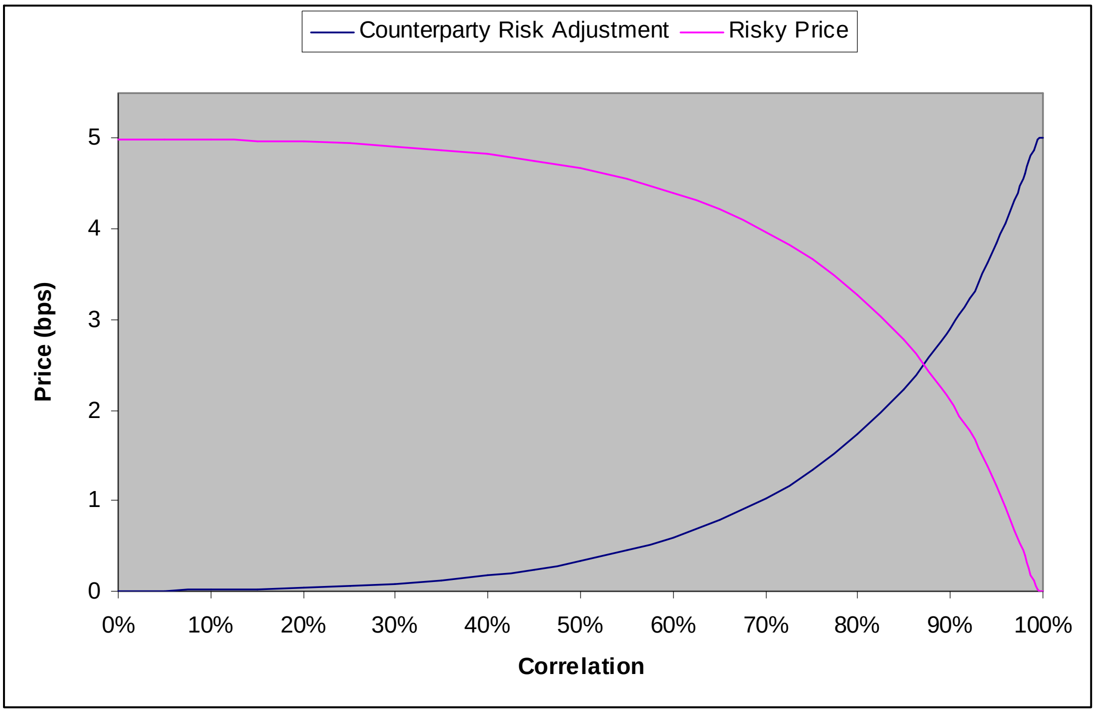
[4] In practice, although perhaps somewhat implausible, the fact that credit ratings and spreads may not move together, and the possibility of ratings arbitrage, may mean that a CDPC or monoline can recover from a restricted state without taking losses or raising additional capital.

small and as such risks diversify reasonably well across asset classes, such risk may be handled as primarily insurance risk and not heavily risk managed[5].

A buyer of senior credit protection from a credit insurer may apply the above argument that their position is equivalent to fully collateralised protection less a counterparty risk adjustment which is small due to the triple-A nature of the credit insurer. However, it is not the absolute rating that is important which can be illustrated with a simple example. Suppose one buys in uncollateralized form a European digital contract that pays a unit notional at time T if some binary event denoted by B has occurred. The value of this contract with zero interest rates is $V(t) = E[B]$. Denoting the counterparty default time by τ and assuming zero recovery, the risky payoff is $\tilde{V}(t) = V(t) - E[1_{\tau<T} B]$. Now, assume the counterparty has an underlying default probability of 10 bps and that the risk-free value of the digtal is 5 bp: the value of this risky contract is 5 bps minus the joint probability of exercise of the contract and default of the counterparty. With a simple Gaussian structure[6] we obtain the results shown in Figure 3. At any “reasonable” level of correlation there is only a small counterparty risk adjustment but at a high correlation the contract value can be significantly less and eventually worthless.

Figure 3. Simple example illustrating impact of correlation on counterparty risk.



We illustrate another example in the Appendix corresponding to the pricing of an out of the money put option.

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[5] Although we have used a risk-neutral measure in equation (1), if the risk were not hedged then it would be more appropriate to use the objective measure. In practice, desks seem to hedge some large risks and leave others unhedged.

[6] In this case the joint probability required is given by a bivariate Gaussian cdf $\Phi\left(N^{-1}(10bps), N^{-1}(5bps), \rho\right)$ where $N(.)$ is the univariate Gaussian cdf and ρ is the correlation.

Simplified CDPC or Monoline

We will focus solely on the value of the protection leg of a tranche since this is the key component in the analysis. We start with the stylised assumption that the credit insurer has a static leverage and furthermore allocates capital on a pro-rata basis to each of its counterparties. This basically means that we assume a protection buyer on a tranche covering losses in the range $[A, B]$ has a claim on at least a certain amount of "available collateral" (as in the LSS case) which we denote by $\alpha(< B - A)$. We shall argue later that the general case involving multiple counterparts simply involves the value of α being a hidden random variable.

We denote the fully collateralised value of the underlying tranche at time t by $V_{A,B}(t)$. The counterparty risk occurs as a result of the fact that the protection buyer has only a sure claim on the available collateral α whereas the full value of protection is $(B - A)$. Given the restricted operating state, before losses hit the tranche, there may be some mitigating action. The counterparty risk is characterised by $V_{A,B}(t) > \alpha$ where the mark to market of the tranche (potentially including losses) is greater than the available collateral. The protection buyer is therefore short an option with strike α referenced to $V_{A,B}(t)$ with payoff $\left(V_{A,B}(t) - \alpha\right)_+$.

From the point of view of the protection buyer, the following outcomes are relevant and lead to some payoff: -

The tranche suffers losses before any unwind or restructuring of the trade: -

i. Losses occur without a change in capital structure of the company.
ii. Losses occur after the company de-leverages via receiving some additional capital or unwinding other trades.

The tranche is unwound at a time τ (presumably when the credit insurer is in a restricted or termination state).

iii. The company can settle the mark-to-market in full since $V_{A,B}(\tau) \leq \alpha$.
iv. The company cannot settle the mark-to-market in full since $V_{A,B}(\tau) > \alpha$.

The value of the protection leg of a CDO with maturity T at time t can be written (for example Laurent and Gregory [2005]) as: -

$$V_{A,B}(t) = E^Q\left[\int_t^T B(t,s)\,dM_{A,B}(s)\right], \qquad (2)$$

where $M_{A,B}(s)$ is the cumulative tranche loss and $B(t,s)$ represents the risk-free discount factor at time s. With the four scenarios above, we can generalise equation (2) for the value of protection purchased from a CDPC or monoline with effective collateral of α as: -

$$
\begin{aligned}
\tilde{V}_{A,B,\alpha}(t) \quad &= E\left[1_{\tau>s}\int_t^T B(t,s)dM_{A,A+\alpha}(s)\right] && \text{Wind-down value (i)}\\
&+ E\left[1_{\tau>s}\int_t^T B(t,s)dM_{A+\alpha,B}(s)\right] && \text{De-leverage value (ii)}\\
&+ E\left[1_{\tau<T} B(t,\tau)V_{A,B}(\tau)\right] && \text{Clean unwind value (iii)}\\
&- E[1_{\tau<T} B(t,\tau)(V_{A,B}(\tau)-\alpha)_+] && \text{Counterparty risk (iv)} \qquad (3)
\end{aligned}
$$

This is similar to the pricing formula for a LSS structure given in Gregory [2008].

Pricing Bounds

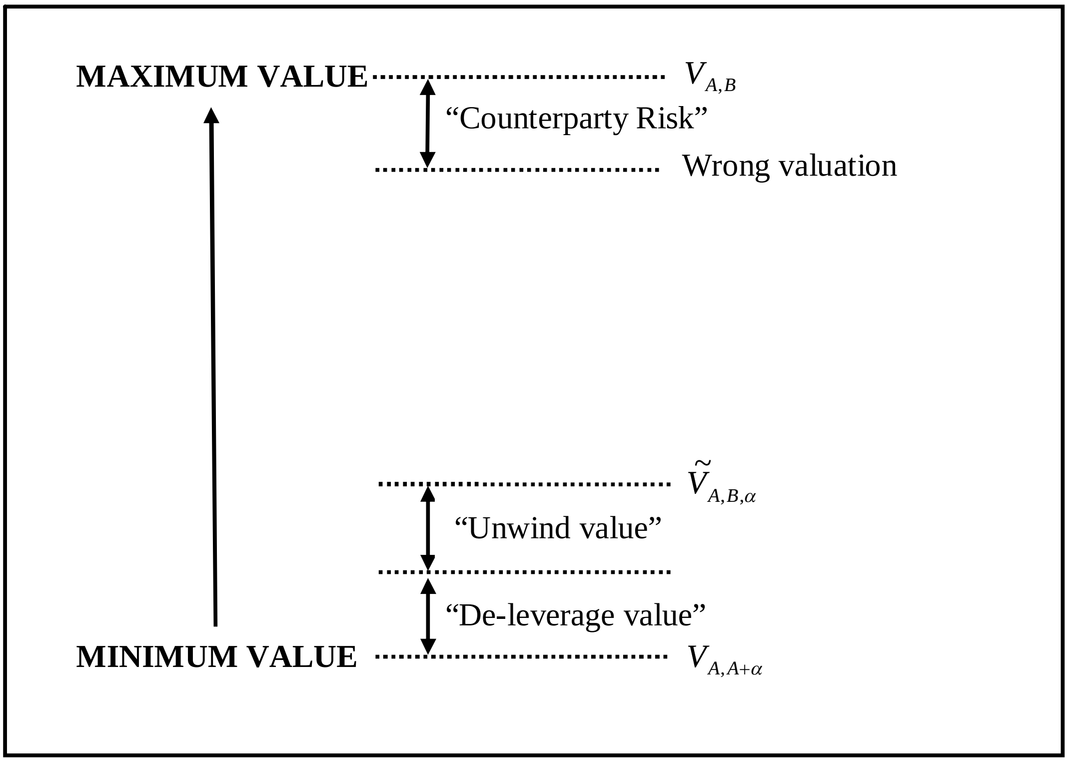
The above equation shows that the bounds on the value of protection are $V_{A,A+\alpha}(t) \le \tilde{V}_{A,B,\alpha}(t) \le V_{A,B}(t)$. The minimum value for the protection corresponds to assuming that there is no chance of the company de-leveraging or unwinding the trade, whilst the upper bound would require the assumption of zero counterparty risk. These bounds will be wide since the credit insurer will be rather highly leveraged.

Obviously, the protection buyer would like to argue that the value is close to the upper bound. However, returning to the value of protection and combining the third and fourth terms we obtain: -

$$
\tilde{V}_{A,B,\alpha} = E\left[1_{\tau>s}\int_t^T B(t,s)dM_{A,A+\alpha}(s)\right] + E\left[1_{\tau>s}\int_t^T B(t,s)dM_{A+\alpha,B}(s)\right] + E\left[1_{\tau<T} B(t,\tau)\min\left(V_{A,B}(\tau),\alpha\right)\right] \qquad (4)
$$

Rather than having a value of protection less some counterparty risk, the buyer of protection has a rather complex set of payoffs related to the precise structure and operating environment of the credit insurer. The last two terms clearly represent a challenge to value even in this rather simplified example. The situation is illustrated schematically in Figure 4.

Figure 4. Illustration of valuation approaches for protection purchased from a CDPC or monoline insurer under simplified assumptions of executing a single trade.



De-leverage and unwind value

In the simplified case we can argue that the value of the tranched protection is worth at least the value of a fully collateralised $[A, A+\alpha]$ tranche. Although, due to the high leverage of the company, this will be considerably less than full $[A, B]$ protection. The question is whether the protection buyer can claim that any value arises from the other mechanisms in which they may receive the value of future losses. This could arise from a de-leverage of the company by either raising additional capital or restructuring transactions. Whilst it is not possible to make a quantitative assessment, we do note that, in the event that the company is forced into unwinding or restructuring trades (in order to return to normal operation) then it is faced with the following: -

$$\text{Capital before} > \text{Capital after} + \text{MTM gain/loss.} \qquad (5)$$

In other words, any crystallised losses must be smaller than the capital reduction achieved. This is very hard to assess since capital requirements are determined by credit ratings whilst any losses will be defined by market credit spreads. The most likely scenario of ratings lagging spread changes represents the worst case since the company may realise significant mark-to-market losses and be forced into termination since it cannot find a way to restructure or unwind trades in order to reduce the capital requirement sufficiently. Indeed, if equation (5) cannot be satisfied then it creates a "death spiral" due to the crystallised losses being greater than the associated capital relief.

To assess the value represented by potential de-leverage, unwind or indeed recapitalisation is clearly not obvious. However, the point is more that such value cannot obviously be monetarized by the protection buyer other than as a windfall gain.

The Multiple Counterparty Case

In reality a protection buyer will not have a claim on a specific amount of collateral and the effective leverage will be determined not only by the leverage of the credit insurer but also by the characteristics of the trades vis à vis the other protection sold. The true value of α that a protection buyer sees will differ since claims would be settled acccording to the timing of losses. Indeed, there is likely to be significant heterogeneity over any unwound or restructured trades depending on the mark-to-market and marginal capital requirements in question.

In the previous analysis, the leverage seen by a given counterparty now becomes α^*, a hidden random variable, and it is rather trivial to argue that the lower bound in this general case is zero. This is not an over-dramatic claim : for example, a relatively senior tranche with good ratings stability can be argued to represent zero value for all of the terms in equation (4) since it will never be optimal for the credit insurer to unwind the trade nor will it be near the "front of the queue" to receive settled losses.

Let us briefly consider how a protection buyer might "enhance" the value of their protection. Since there are multiple counterparties, in order to maximize the value of protection on a given tranche, a protection buyer needs to increase the value of the second and third terms on the right of equation (4). This could be achieved in a variety of (sometimes perverse) ways. For example: -

- Structure the tranche so it is likely to be downgraded (to create incentive for unwind).
- Create a tranche with a negative forward value, for example by paying a step-up coupon.
- Ensure the tranche has smaller mark-to-market volatility (to reduce the chance of large losses), for example having a shorter maturity.
- Make the tranche sufficiently junior so that in a wind-down / run-off state it will be one of the first to take losses whilst there is still available capital.

Since the other counterparties may have been aware of these issues also, the buyer of protection only has real value if they can convincingly argue that they are at the "front-of-the-queue" in their claim on the relatively small amount of capital.

A very important point to be made is that when a CDPC goes into a wind-down mode or a monoline into runoff, the protection buyer loses all potential value from future unwind or de-leverage and thus the idea of a rating becomes somewhat immaterial. Yet, for example, a CDPC may still be rated triple-A partially by virtue of the fact that the assessment of losses under the objective measure by the rating agency is still within the relevant thresholds. Such assessment will be of little consolation to a swap counterparty who bought protection on a $[A, B]$ tranche and subsequently has to recognise that they effectively have only $[A, A+\alpha^*]$ protection where α^* is a hidden random variable that effectively defines their place "in the queue".

Conclusion

In this article we have presented a quantitative analysis of the pricing of senior tranched protection from a CDPC or monoline insurer. We have argued that a traditional approach of assessing counterparty risk is highly questionable. By making some parallels with the similar LSS structure, we have outlined a more rigorous pricing of senior protection which suggests the protection is of very limited value which should lead to some unpleasant surprises for parties trading with CDPCs or monolines.

Having specialised triple-A companies as providers of super senior credit protection may represent an advance in terms of efficient credit risk transfer. However, given the systemic nature of senior credit risk, it is critical that these companies have solid foundations. The sub-prime crisis of 2007 has resulted in losses for monolines themselves but also for counterparties with some of the significant writedowns by banks being a result of credit valuation adjustment related to hedges with financial guarantors. The theoretical idea presented here that senior protection purchased from monolines or CDPCs is close to worthless may be proven already.

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Appendix : The Put Option Analogy

You would think twice about buying an OTC put option from a company on its own stock. You would even question buying such an option from a counterparty highly correlated to the underlying. The more out of the money the put option, the more concerned you would be (since the stock has to decline further to be in the money). Let us illustrate this with a simple example. We define the terminal stock price S_T as: -

$$S(T) = S(0)\exp\left((r - q - \sigma^2 / 2)T + \sigma\sqrt{T}V_S\right),$$

with the usual parameters of $S(0)$ the initial price, r the risk-free interest rate, q the dividend yield, σ the volatility, T the option maturity and V_S a standard Gaussian variable. Next we define a single default process by: -

$$\tau = G^{-1}\left(\Phi(V_d)\right),$$

where $G(.)$ is the cumulative default probability, τ the default time, V_d a standard Gaussian variable and $\Phi(.)$ the cumulative standard normal distribution function. Let us correlate the default and equity returns via: -

$$V_S = \sqrt{\rho}V + \sqrt{1-\rho}\tilde{V}_S \qquad V_d = \sqrt{\rho}V + \sqrt{1-\rho}\tilde{V}_d,$$

where ρ is the correlation and $\tilde{V}_S$ and $\tilde{V}_d$ are independent standard Gaussian variables. The payoff of a default and out put option is $\left(K - S(T)\right)_+ 1_{\tau > T}$. We can derive a modfied Black-Scholes formula[7] for the price of this contract as: -

$$P = e^{-rT}\left(- F.A_1 + K.A_1\right)$$

$$A_1 = \int_{-\infty}^{\infty}\Phi\left(-\frac{\sqrt{\rho}u + \sigma\sqrt{T} + d_2}{\sqrt{1-\rho}}\right)\Phi\left(\frac{\sqrt{\rho}u + \rho\sigma\sqrt{T} - \Phi^{-1}\left(G(T)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du$$

$$A_2 = \int_{-\infty}^{\infty}\Phi\left(-\frac{\sqrt{\rho}u + d_2}{\sqrt{1-\rho}}\right)\Phi\left(\frac{\sqrt{\rho}u - \Phi^{-1}\left(G(T)\right)}{\sqrt{1-\rho}}\right)\varphi(u)du$$

$$d_2 = \left[\ln(F / K) - \sigma^2 / 2)T\right] / \sigma\sqrt{T}$$

where $F = e^{(r-q)T}$ is the forward price of the stock and $\varphi(u)$ is the standard normal density function.

Taking a simple example we price a default put option with parameters: $S_0 = 100$, $r = 4\%$, $q = 2\%$, $\sigma = 45\%$, $T = 1$, $\rho = 50\%$, $G(1) = 1\%$. We will price the following two options.

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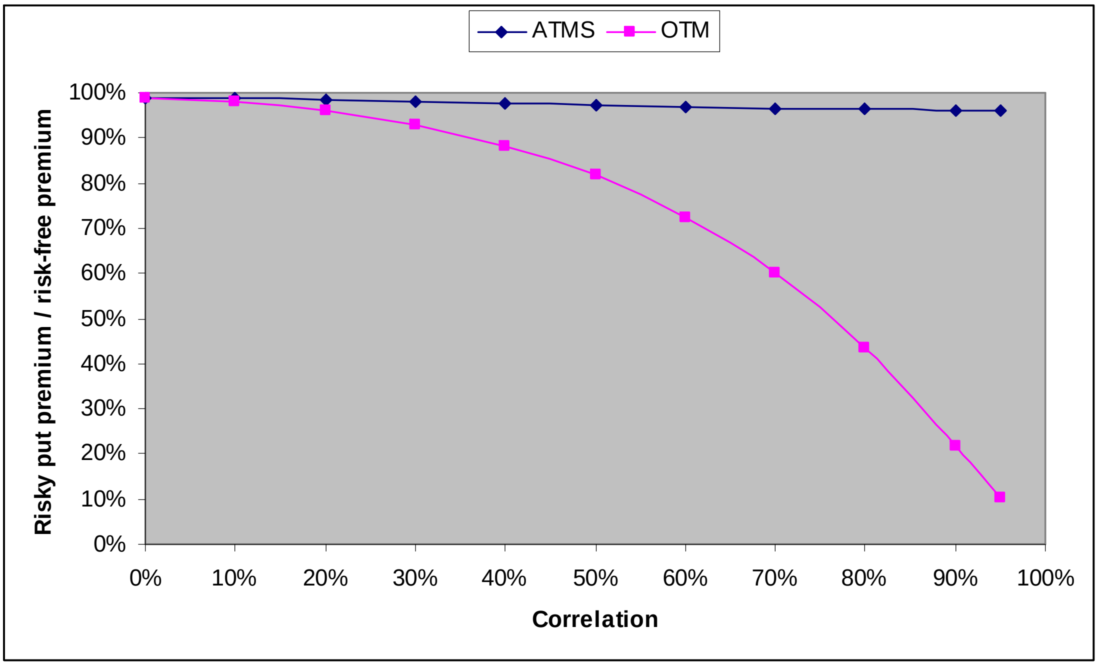
[7] A derivation and implementation of this formula is available from the author on request.

i) At the money strike (ATMS) put option, $K = 100$.

ii) Out of the money (OTM) put option, $K = 30$.

The first option has an exercise probability of 57.4% whilst the second (which is similar to an equity default swap) just 0.64%. This constitutes a probability more akin to senior tranches in the structured credit world. Pricing these two default and out options as a function of correlation gives the results shown in Figure 5. Whilst the ATMS option has little sensitivity, for the OTM option the correlation impact is huge.

Figure 5. Price of a default and out (risky put option) with respect to the risk-free value as a function of correlation.



Clearly this approach is very simplistic with no representation of volatility smile but it illustrates the basic point. A senior tranche is similar to the OTM put option in terms of the relative likelihood of exercise. You wouldn't buy such an OTM option from a highly correlated counterparty so why would we buy credit protection under similar circumstances?