Marginally Conservative?

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Central counterparties (CCPs) require both initial margin and default fund contributions in case of a clearing member default. Initial margin requirements are set at a conservative level so as to minimise the likelihood of mutualising losses into the default fund in a default scenario. However, this article shows that such conservative initial margins can be problematic. Firstly, a larger initial margin requirements may or may not increase the exposure of a clearing member to the CCP. Secondly, high initial margins can have adverse consequences and expose a central counterparty to greater losses in an auction. This suggests that larger default funds and heterogeneous loss allocation (where aggressive auction bidding is rewarded) and lower initial margin requirements may be preferable in some situations.

1. Introduction

In the aftermath of the financial crisis from 2007 onwards, policymakers embarked on a number of regulatory changes largely aimed at moving risk away from global investment banks, and the bilateral OTC derivatives market. This was driven by the general view that the size, opacity and interconnectedness of the OTC derivatives market were too significant. One important such reform was that of mandatory central clearing requiring all standardised OTC derivatives (such as interest rate and credit default swaps), to be cleared through central counterparties (CCPs).

CCPs are not a new idea and have been a part of the derivatives landscape for well over a century in connection with exchange-traded products. On derivatives exchanges, central clearing gradually developed as a homogenising feature that was relevant given the anonymity of trading that an exchange facilitates. OTC derivatives clearing began in 1999 with LCH.Clearnet's SwapClear service. Clearing of OTC derivatives operates differently to that of exchange-traded products since the underlying contracts are still executed bilaterally. Furthermore, the fact that OTC derivatives are more illiquid and longer-dated than exchange-traded ones makes "OTC clearing" more complex. Since the OTC derivatives market is also around an order of magnitude larger than the corresponding exchange-traded market (in terms of total notional amount outstanding, notwithstanding the longer-dated nature of OTC products), the topic of OTC clearing requires significant attention to understand the risk management mechanism of a CCP. In particular, the auction mechanism (where a CCP seeks to replace contracts in the aftermath of a default), may be worthy of particular analysis given the relative illiquidity, size and complexity of OTC derivatives.

The last few years have seen a significant amount of effort put into characterising counterparty credit risk and related components such as funding and capital in bilateral markets. Terms such as CVA (credit value adjustment), FVA (funding value adjustment) and even KVA (capital value adjustment) are relatively well characterised in mathematical terms with a large body of research having been developed (see, for example, see Morini and Prampolini 2011, Hull and White 2012, Burgard and Kjaer 2013, and Green and Kenyon 2014). In contrast, there has been less mathematical research around the assessment of risk within CCPs, some obvious exceptions being and Gai and Kapadia (2010), Duffie and Zhu (2011) and Arnsdorf (2014). A review of risk management approaches of CCPs prior to most OTC clearing is given by Knott and Mills (2002).

Regulatory reform will require significantly more OTC clearing than seen in the past. "OTC CCPs" will be perhaps the largest of all systemically important financial institutions. Given the significance of OTC CCPs as a result of the clearing mandate, there will be a need to understand the risk that CCPs represent and to build a framework around assessment of CCP exposure and costs similar to that which exists around CVA/FVA/KVA in bilateral markets. The aim of this article is to provide some first steps in this direction.

2. Exposure to a CCP

A first obvious question to ask is how to characterise the exposure to a CCP from the point of view of one of their clearing members (CMs). Doing this is less straightforward than looking at a normal exposure arising from the potential default of a counterparty such as a financial institution, corporation or sovereign. This is because it is possible to experience a loss as a CCP member in a variety of ways of varying severity. The primary reason for this is that CMs contribute two different forms of financial resources to a CCP: namely, initial margin (IM) and default fund (DF). Furthermore, in extreme circumstances, CMs can be exposed to losses beyond the level of their IM and DF contributions via other loss allocation methods.

In order to understand how the exposure to a CCP arises, it is useful to define the general waterfall that is used to absorb losses in event of one or more CMs defaulting:

- *Initial margin (defaulter).*
- *Default fund (defaulter).*
- *CCP equity contribution ("skin in the game").*
- *Remaining default fund (all clearing members).*
- *Other loss allocation methods.*
- *CCP failure or external support.*

The above shows that a surviving CM can lose their default fund contribution and be exposed to other loss allocation methods without the CCP failing. On the other hand, the initial margin cannot be used by the CCP to allocate losses and is therefore only at risk in the event of CCP failure. Even then it might be argued that initial margin not at risk to a CCP failure since it may be segregated via a third party custodian. However, at the current time CMs generally view their initial margins as at risk to the CCP default either due to the lack of clear legal opinion or for other reasons. Segregation of IM is a moot point in any case since a CM can easily make losses equal to, or even in excess of, their IM contribution via the other loss allocation methods. References to IM exposure below may therefore be literal or may be considered a proxy for exposure to other loss allocation methods. We do assume that losses will be capped at a given level. Whilst certain loss allocation methods can, in theory, lead to unlimited losses it seems reasonable to assume that there is some upper limit to the losses that a CCP may impose.

Let us formalise the loss allocation process in more detail. We consider the exposure of the CMs under a single default[1] of CM 1. The terms IM_i and DF_i represent the initial margin and default fund contributions of a given CM i. Note that we do not here consider explicitly potential additional contributions to the default fund (so-called "rights of assessment") or other loss allocation methods which can all be considered as part of the assumed default fund. Note also that the IM exposure may be viewed more loosely as representing the exposure to other loss allocations methods such as variation margin gains haircutting (discussed below).

With the loss faced by the CCP being denoted by L and the total default funds and initial margins held by the CCP being $DF_T = \sum_i DF_i$ and $IM_T = \sum_i IM$, the total exposure (E) for the surviving CMs in terms of their default fund contributions is:

$$E^{DF} = min((L-B)^+, C-B) \text{ and } E^{IM} = min((L-C)^+, D-C) \qquad (1.)$$

$$B = IM_1 + DF_1\,, C = IM_1 + DF_T, D = IM_T + DF_T$$

The DF exposure arises when losses are above the DP resources $(IM_1 + DF_1)$ but capped at the total default fund and defaulters IM $(IM_1 + DF_T)$. The initial margin exposure begins when the default fund is exhausted and is capped at the total default fund and initial margin $(IM_T + DF_T)$. Assume that the default scenario losses above some threshold $K(< B)$ follow a (normalised) Pareto distribution $f(L) = q(\alpha-1)K^{\alpha-1}L^{-\alpha}$ where q represents the probability of being above the threshold. Arnsdorf (2014) considers the potential parametrisation of this extreme value distribution in more detail. A depiction of the above exposures is shown in Figure 1.

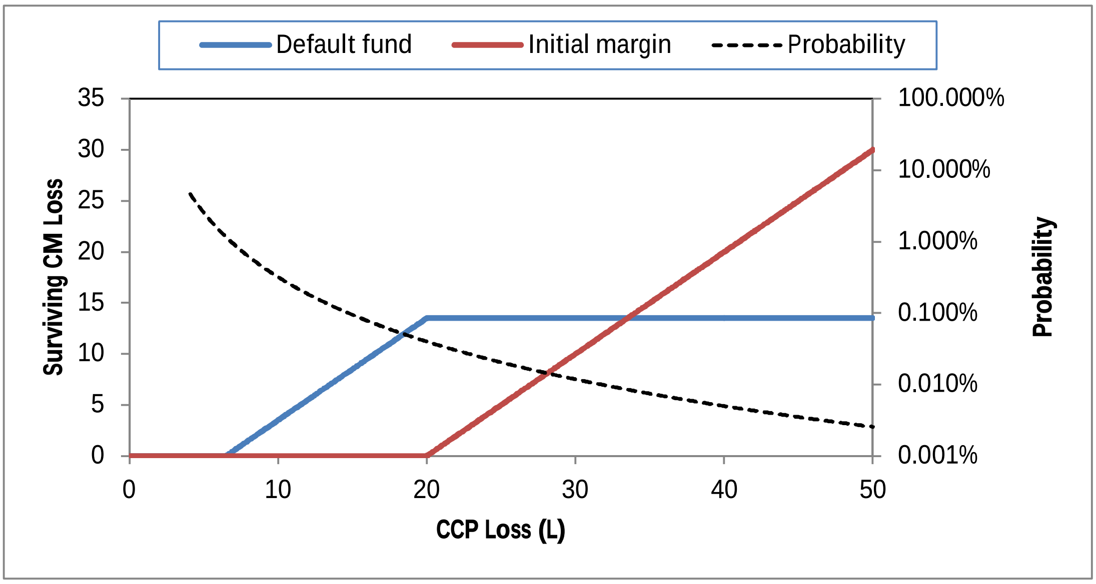

***Figure 1.** Representation of CM default fund (E^{DF}) and initial margin (E^{IM}) exposures (sum of all CMs) as a function of the total CCP loss. Parameters used:* $IM_1 = 5, DF_1 = 1.5, n = 10, \alpha = 3.0, K = 4, q = 100\%$,.

[1] Note that it is possible for CCP losses to also occur under non-default scenarios (e.g. investment losses) but these are generally assumed to be less likely.

Under the above assumptions, the expected or average exposure (EE)[2] for the default fund contribution is:

$$EE^{DF} = \int_{B}^{C} (L - B) f(L) dL + C \int_{C}^{\infty} f(L)\, dL \tag{2.}$$

Which can be shown to be:

$$EE^{DF} = q \left[\frac{C^{2-\alpha} - B^{2-\alpha}}{2 - \alpha} \right] K^{\alpha - 1} \tag{3.}$$

Similarly, the exposure from the IM is:

$$EE^{IM} = q \left[\frac{D^{2-\alpha} - C^{2-\alpha}}{2 - \alpha} \right] K^{\alpha - 1} \tag{4.}$$

The above exposures will be divided between clearing members in a heterogeneous fashion and the relative DF and IM contributions for a given member will also differ (although they will likely be broadly similar). In the below analysis, we consider only the total exposure of all clearing members.

Assuming PFE is defined by a quantile $\gamma(< q)$ of the distribution then the overall PFE of the DF and IM contributions will be:

$$PFE^{\gamma} = min\left((\lambda_{\gamma} - B)^{+}, C - B \right) + min\left((\lambda_{\gamma} - C)^{+}, D - C \right) \tag{5.}$$

Where $\lambda_{\gamma} = Kexp[ln(1-\gamma)/q(1-\alpha)]$ represents the worse case loss at this confidence level. Now suppose the CCP has some target level of "default pays" (DP) resources given by $\Theta_{DP} = DF_T + IM_1$. This is broadly[3] in line with a so-called cover-1 requirement where the CCP default fund covers (at a minimum) the default of the largest clearing member and captures the fact that higher initial margins may lead to smaller default funds. The default fund contribution for all CMs will be $DF_T = (\Theta_{DP} - IM_1)$. The total contribution by all CMs member is then $IM_T + DF_T = \Theta_{DP} + IM_T - IM_1$: this shows that IM is expensive since it is not mutualised.

Under the above cover-1 assumptions, Figure 2 shows the total EE $\left(EE^{DF} + EE^{IM}\right)$ and PFE (PFE^{γ}) as a function of increasing IM levels for given parameter values. Note that the EE is monotonically decreasing with increasing IM which is due to the fact that the overall DP resources increase thereby making a loss less likely even though the total exposure increases (EL^{DF} decreases whereas EL^{IM} increases with the former term dominating). On the other hand, the PFE shows different behaviour, initial increasing due to the higher exposure when posting more initial margin and then decreasing due to the greater DP resources of the CCP. The PFE shape also depends on the confidence level chosen.

[2] Sometimes referred to as EPE (expected positive exposure).

[3] This is not intended to capture precisely the financial resources held by the CCP but just the balance between IM and DF contributions.

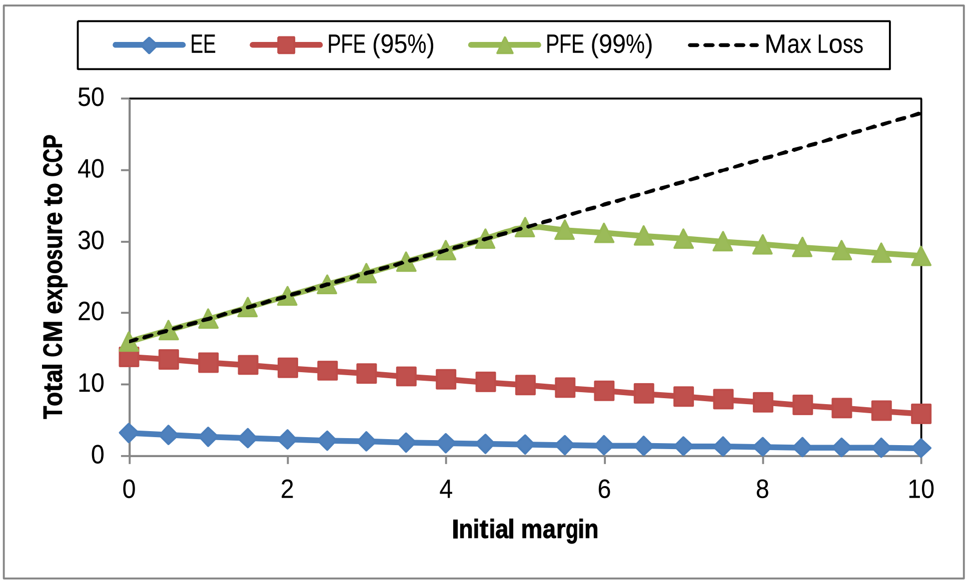

Figure 2. *Comparison of expected exposure (EE) and potential future exposure (PFE) for the surviving CMs as a function of the initial margin contribution of a single CM. Parameters used:* $\Theta_{DP} = 20, n = 5, \alpha = 3.0, q = 100\%, K = 4$.

The above illustrates a paradox with respect to the size of initial margins taken by a CCP. On the one hand, higher initial margins may reduce total exposure since they increase the defaulter pays resources but on the other hand they increase the total exposure to a CCP. Unlike traditional assessment of the exposure to a counterparty, assessing the exposure to a CCP is more difficult since it is not driven by a single default event but instead by IM and DF type exposures which have very different roles and underlying loss probabilities.

3. Auction mechanics

A key point for a CCP clearing significant OTC derivatives is to be able to manage a default of a major CM. Such a CM will clear a relatively large OTC portfolio (and may also clear for a number of clients) and their default will likely create a significant dislocation in market prices. In the aftermath of this default, the CCP must be able to return to a "matched book" by dealing with all of the house and client transactions of the defaulter preferably without having to impose losses on the surviving clearing members, shareholders or even (as in the case of a government bailout) the taxpayer. For more illiquid OTC derivatives, the CCP auction is likely to be a very important mechanism since such products are more illiquid and portfolios harder to macro-hedge (for example, credit default swaps (CDSs), swaptions and inflation swaps).[4] Furthermore, all CMs will see the portfolio in question which may make it even harder for the winning bidding to hedge the resulting position. An interesting point is therefore to consider the dynamics that might occur around such an OTC auction of this type.

We consider the aftermath of the default of one (or more) clearing members and assume that a CCP is holding an auction of a certain class of OTC products and has certain financial resources to absorb the underlying losses. Prior to the auction, the CCP will have likely macro-hedged the portfolio in question to neutralise key first order sensitivities but for some products this will difficult and potentially impractical (for example, for CDS indices there are no more liquid

[4] See, for example, "LCH warns of CCP auction risk", Risk, 6th April 2015.

macro-hedges such as CDS futures contracts). The below analysis considers the impact after any such macro-hedging.

Assume that CM 1 will default and that the remaining $(n-1)$ CMs will bid amounts $V_{MTM} - x_i$ in the auction where V_{MTM} is the current mid mark-to-market value of the relevant portfolio and x_i represents some premium priced in by each CM. The x_i can be seen as a charge for taking the portfolio away from the CCP and will be increasingly positive as the CM is less inclined to take such a portfolio. The winning bidder will be denoted by W with $x_W = \min(x_2, \ldots . x_n)$ and will charge the least for taking on the risk. When bidding in the auction, the CMs are exposed to market volatility and hedging costs on the portfolio resulting in a final value of $x_i + \varepsilon_i$, where ε_i is assumed to be a Gaussian variable with mean and standard deviation μ_i and σ_i. CMs will choose x_i such that this final position is very likely to be positive for them but a very negative ε_i can expose them to losses. This includes any loss from the assumed initial value of the (macro hedged) portfolio (V_{MTM}). The random term ε_i may contain components shared by all CMs (such as market volatility) but may also be driven by components specific to a given CM (such as their ability to hedge a portfolio in an efficient and timely fashion). The risk appetite of each CM will clearly also be relevant.

The costs from the auction can be absorbed by the DP resources of the CCP (Θ_{DP}) as above. If the DP resources are not sufficient to absorb the auction loss then they will be mutualised via the default fund and other methods. Assume that clearing member *i* will suffer a pre-defined loss fraction of α_i^W of the total DP resources if winning the auction and an alternative loss fraction $\alpha_i^L (> \alpha_i^W)$ otherwise with $\sum_i \alpha_i^W I(x_i = x_W) + \sum_i \alpha_i^L I(x_i \neq x_W) = 1$. The overall loss for a given CM resulting from the default fund exposure will be:

$$\alpha_i^W [x_i - \theta_{DP}]^+ I(x_i = x_W) + \alpha_i^L [x_W - \theta_{DP}]^+ I(x_i \neq x_W) \qquad (6.)$$

In reality, a number of different loss allocation methods may be used that will correspond to different expected values of α_i^L and α_i^W. A brief outline is given below (for more details on these loss allocation methods see Elliott 2013).

- *Homogenous default fund utilisation and rights of assessment.* This corresponds to absorbing losses approximately pro-rata with respect to the size of the position each CM has via current and additional default funds contributed. As such, a CM would not expect their behaviour in the auction to impact their losses $(\alpha_i^L = \alpha_i^W)$.
- *Default fund tranching.* Some CCPs may allocate default fund losses in tranches based on the bidding behaviour in the auction (for example LCH.Clearnet has the concept of Auction Incentive Pools[5]). This would lead to $\alpha_i^L \geq \alpha_i^W$.
- *Bidding penalties.* This involves penalising bids in the auction that are considered outside a reasonable range and would also lead to $\alpha_i^L \geq \alpha_i^W$.
- *Variation margin gains haircutting (VMGH).* This corresponds to haircutting the values of variation margin owed by the CCP. This may be fairly arbitrary and therefore have a rather unclear and arbitrary impact on α_i^L and α_i^W.
- *Partial tear-up.* This involves the CCP terminating opposite positions to those in the defaulter's portfolio and like VMGH will have an uncertain outcome.

[5] See LCH.Clearnet Ltd Default rules section 2.4, available at www.lchclearnet.com.

- *Forced allocation.* In extreme cases, a CCP may force allocate a portfolio to a CM at a defined price. The CM could be chosen, for example, as the worst bidder in the auction. This clearly leads to and $\alpha_{i=j}^{L} = 1$, $\alpha_{i}^{W} = \alpha_{i\neq j}^{L} = 0$ where *j* denotes the worst bidder.

The analysis below will assume that CMs will have access to various quantities such as the DP resources of the CCP. This is obviously not completely true in practice but since there is growing transparency with respect to initial margin and default fund sizing methodologies of CCPs, it is not unreasonable to suggest that CMs would be able to make a reasonable estimate of these quantities.

Assuming that α_i^L and α_i^W are known, the total profit and loss for a given CM *i* as a result of the auction would therefore be:

$$W_i = \left((x_i + \varepsilon_i) - \alpha_i^W [x_i - \theta_{DP}]^+\right) I(x_i = x_w) - \alpha_i^L [x_W - \theta_{DP}]^+ I(x_i > x_w) \qquad (7.)$$

If the clearing member makes their bid more aggressive then they obviously will increase their chance of winner the auction and make default fund losses less likely. However, they will also increase their chance of making losses as a result of the risk associated with the portfolio. Under the assumption of exponential utility, $U(y) = -\exp(-\lambda y)$, the expected utility of the i^{th} CM will be:

$$E[U(W_i)] = -\exp\left(\left(-\lambda_i(x_i + \mu_i) + \frac{\lambda_i^2 \sigma_i^2}{2} + \alpha_i^W \lambda_i [x_i - \theta_{DP}]^+\right) I(x_i = x_W)\right) \qquad (8.)$$
$$\times \exp\left(\lambda_i \alpha_i^L [x_W - \theta_{DP}]^+ I(x_i > x_W)\right)$$

A first observation is that if CMs co-operate to maximise their total utility then they will ensure that $x_W \geq \theta_{DP}$ since otherwise the winning bidder will take on the portfolio at a worse price but no clearing member will see the benefit of this in the loss allocation process since any excess θ_{DP} will be effectively returned to the bankruptcy estate of the defaulted member.

Of course, CMs will not be able to collude in the auction and any such behaviour will be discouraged by the operational process and is likely to be illegal. However, given the CMs may have a reasonable idea of each other's portfolios and risk tolerance, some form of co-operative behaviour is inevitable. The results below assume perfect co-operation although this does not change the general conclusion.

Suppose that CM *k* has the most favourable risk aversion and hedging parameters and will therefore win the auction under optimal (co-operative) behaviour. The best price they should quote should be the solution to:

$$\left(-\lambda_k(x_k^* + \mu_k) + \frac{\lambda_k^2 \sigma_k^2}{2} + \alpha_k^W \lambda_k [x_k^* - \theta_{DP}]^+\right) = \lambda_k \alpha_k^L [x_k^* - \theta_{DP}]^+, \qquad (9.)$$

since this represents a point where they are indifferent to winning and losing the auction. The price achieved would therefore be:

$$x_k^* = x_W = max\left(\min\left(\frac{\lambda_k\,\sigma_k^2}{2} - \mu_k, \frac{\theta_{DP}\,(\alpha_k^L - \alpha_k^W) + \frac{\lambda_k\,\sigma_k^2}{2} - \mu_k}{1 + (\alpha_k^L - \alpha_k^W)}\right), \theta_{DP}\right) \tag{10.}$$

An illustration of this expression is given in Figure 3 (note that we represent the value x_i which represents the premium charged with respect to the MTM and so a smaller price is more aggressive). If the default fund allocation is not dependent on whether or not *k* wins the auction ($\alpha_k^L = \alpha_k^W$) or if the DP resources are sufficient then the price is based on the expected change in the value of the portfolio (μ_k) and its risk which is represented by a penalty relating to the variance (σ_k^2) and risk aversion coefficient (λ_k) of the CM. However, if the default fund is likely to be utilised then a better price is achieved by increasing α_k^L compared to α_k^W, which is the case in methods such as default fund tranching as discussed above.

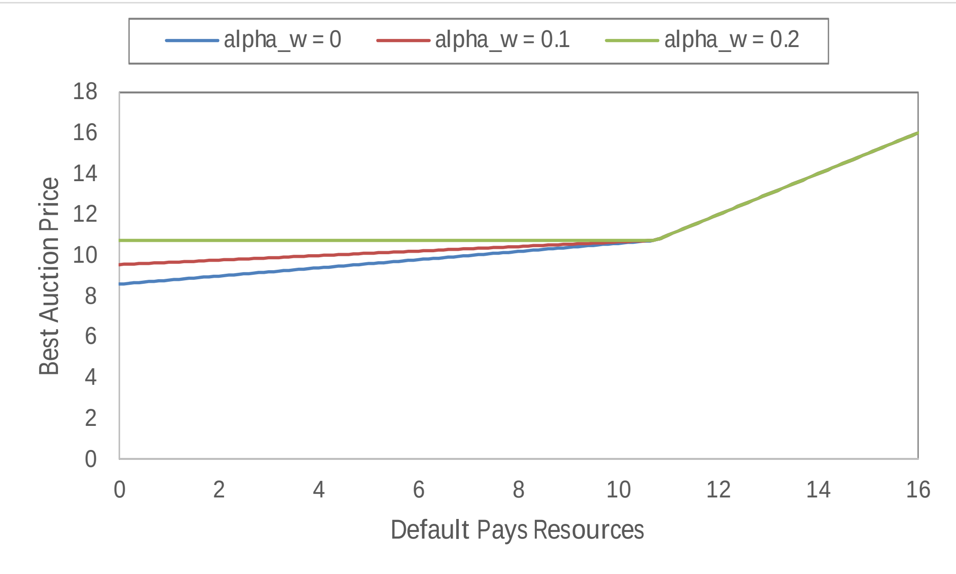

Figure 3. *Best auction price (a lower price is more advantageous to the CCP) achieved as a function of the default pays resources* (θ_{DP}) *for different default fund allocations. Parameters assumed:* $\lambda_k = 0.7$, $\sigma_k = 5$, $\mu_k = -2$, $\alpha_L = (1 - \alpha_W)/4$.

For DP resources that are low enough then heterogeneous loss allocation may be helpful. In such cases, as α^L is increased compared to α^W, then price the CCP achieves is better since the winning bidder is compensated via a lower default fund loss and does not need to achieve this compensation via their price. Note that the best price is achieved when the DP resources are zero and non-homogenous loss allocation is involved. Note also high IMs make heterogeneous loss allocation irrelevant.

However, if the DP resources are too high then heterogeneous loss allocation is worthless in incentivising bidding in an auction since the CMs are not concerned about losses potentially hitting their default fund. Furthermore, under cooperative behaviour the CMs will worsen their prices as the DP resources increases so as to be able to extract the maximum amount of gain from the defaulter's estate. There is some evidence of this behaviour in practice: for example, in the Lehman bankruptcy, there were claims that CME members profited from participating in the auction[6]. Such effects may not be seen as problematic by the CMs, or even the CCP, but will be a significant detriment to other creditors and may therefore be the source of potential systemic risk.

The above suggests that CCPs should utilise heterogeneous loss allocation methods in order to incentivise CMs to bid aggressively in auctions and in turn minimise the CCPs exposure to losses. However, in order for heterogeneous loss allocation to work, the DP resources must not be too large. In setting initial margin (and therefore DP) requirements, CCPs therefore face a dilemma:

- Initial margins should be very conservative so as to make the chance of mutualisation of losses highly unlikely.
- Initial margins should not be so high that CMs think they are easily sufficient and will therefore consider any heterogeneous loss allocation to be irrelevant. Furthermore, they will likely exhibit some co-operative behaviour so as to collectively extract the maximum financial gain from the CCP in a default scenario.

4. Conclusion

This article has described some first ideas in relation to the risk of OTC CCPs. We show that quantifying the exposure that a clearing member has to a CCP is not straightforward due to the different ways in which losses can be experienced. We have also examined the potential dynamics in a CCP auction and their link with the various forms of loss allocation that may occur. We show that heterogeneous loss allocation, where bidding in the auction is rewarded, can be beneficial in incentivising bidding. This suggests that methods that reward aggressive bidding such as default fund tranching may be preferable to more arbitrary methods such as variation margin gains haircutting. This may solve a potential “prisoner’s dilemma” where all clearing members may bid defensively. However, in order for heterogenous loss allocation to work, the “default pays resources” must not be too large. A further problem with large initial margins is that under cooperative assumptions the clearing members will aim to ensure that the auction winner’s bid is adjusted so as to collect as much of the defaulter pays resources as possible. Practically this means that high initial margins can be dangerous as was illustrated with apparent auction windfalls gained in the aftermath of the Lehman Brothers bankruptcy. These findings are in contrast to perceived views that high initial margins contribute solely to making CCPs safer.

[6] For example, see “Firms reaped windfalls in Lehman auction: examiner”, Reuters, April 15th 2010 and “CME, Lehman Book Bidders Likely Protected From Lawsuits”, Wall Street Journal, April 15th 2010. It should be noted that such effects did not always occur, for example SwapClear required only 35% of the total initial margin it held from Lehman (LCH.Clearnet 2011).

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