Gaining From Your Own Default – The Strange Case of DVA

Jon Gregory

www.oftraining.com

Outline

1. Background, accounting rules and examples

2. CVA (credit value adjustment)

3. CVA and capital

4. DVA (debt value adjustment)

5. How to realise DVA

6. DVA and funding

EUROPEAN POLICY FORUM

Counterparty Casino:

The need to address a systemic risk

By Jon Gregory

Background, Accounting Rules and Examples

The Trials of Regulation (I)

- What don't I like as a regulator?

- Different institutions valuing assets differently
  - Institution A trades a derivative with institution B and they both book a profit!
- Institutions making profits based on "mark-to-model"
  - By the time we realize our model was wrong then bonuses have been paid……
- Balance sheets not being a zero sum game
  - For example, if a firm issues a bond do they mark its par value as a liability or its market value?

The Trials of Regulation (II)

- How to solve the problems?

- Different institutions valuing assets differently
  - Mark-to-market (fair value accounting)
- Institutions making profits based on “mark-to-model”
  - Mark-to-market
- Balance sheets not being a zero sum game
  - Mark-to-market (of own liabilities on balance sheet)

Pricing Liabilities With Your Own Credit Risk

- Suppose a firm issues a bond (par value $100) with a treasury like coupon
- The market will only pay $95 for this bond due to the firm's credit risk

Assets	Liabilities
……….	……….
……….	……….
……….	……….
……….	……….
$95 cash	$95 bond

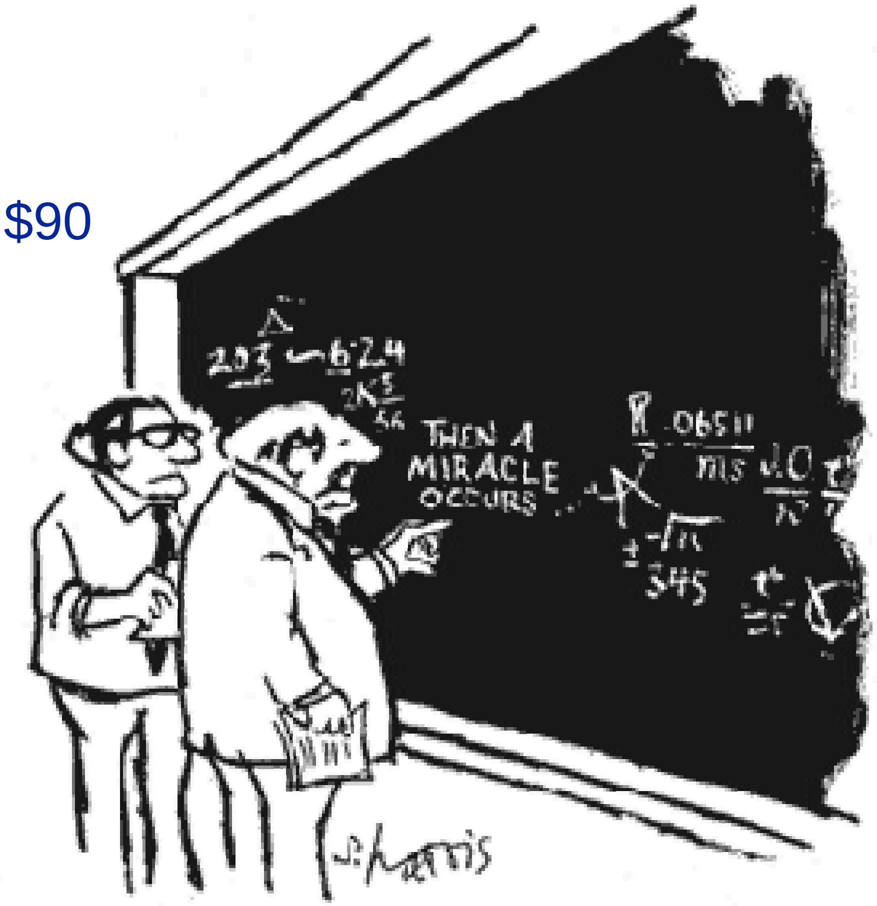
~~Assets~~	~~Liabilities~~
~~……….~~	~~……….~~
~~……….~~	~~……….~~
~~……….~~	~~……….~~
~~……….~~	~~……….~~
~~$95 cash~~	~~$100 bond~~

Gaining from Your Own Default

- The firm's credit spread widens
- The market price of the bond is now $90
- Profit of $5

Assets	Liabilities
……….	……….
……….	……….
……….	……….
……….	……….
$95 cash	$90 bond

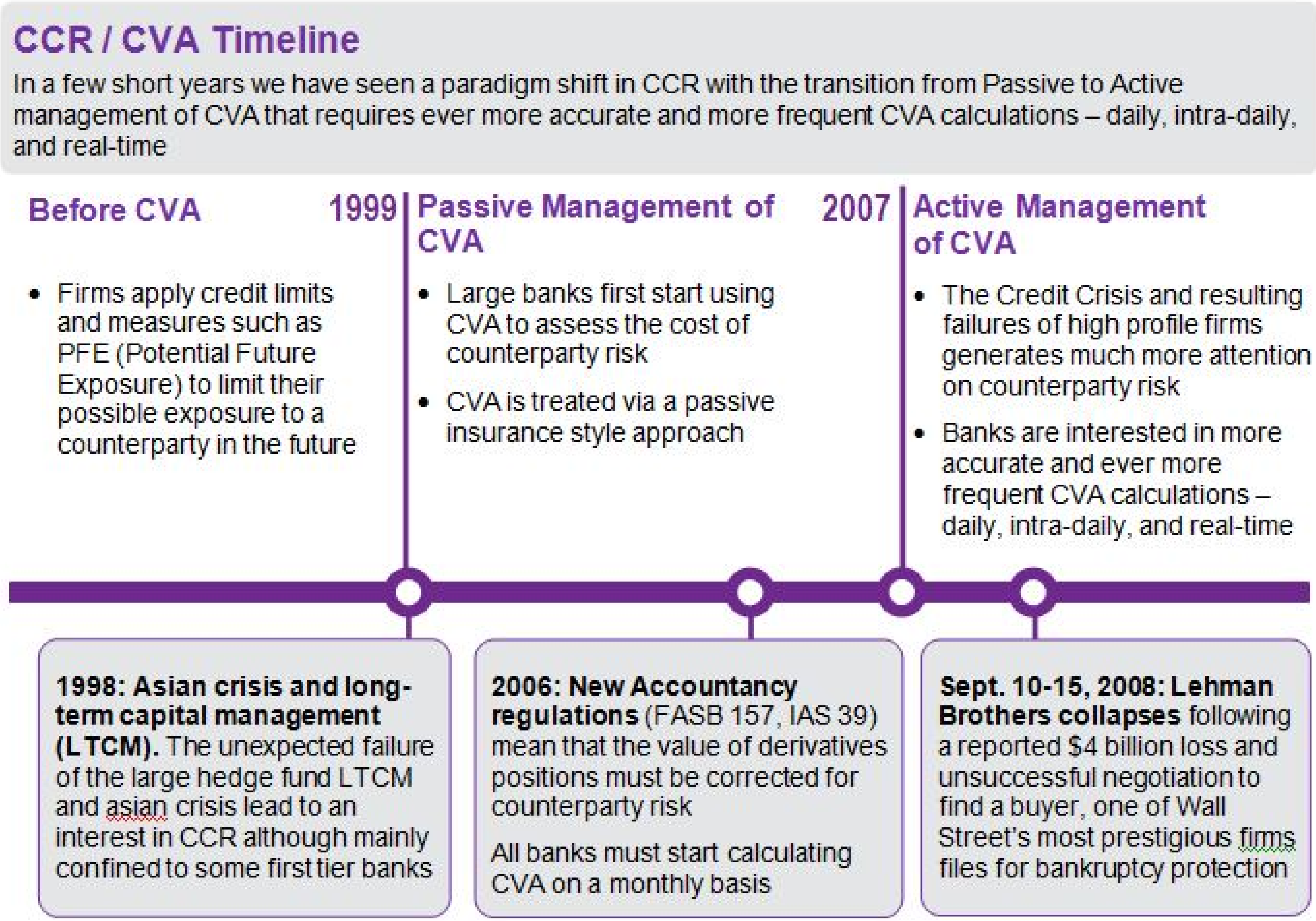
18% of pre-tax income for JPM, MS, BoA and GS in second quarter



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

CVA

History of Counterparty Risk and CVA



Source: Algorithmics

CVA (Credit Value Adjustment)

- CVA is the price of counterparty risk (expected loss) and is a **cost**

$$\text{Risky Derivative} = \text{Derivative} - \text{CVA}$$

- Crucial to be able to separate valuation of derivatives and their CVA (below formula assumes no wrong way risk)

$$CVA(t) = (1-\delta_C)\int_t^T EE(u)\,dPD_C(u)$$

Percentage recovery value

Expected exposure including discounting (how much we expect to lose)

Default probability (how likely is counterparty to default at this time)

But CVA is Very Complex

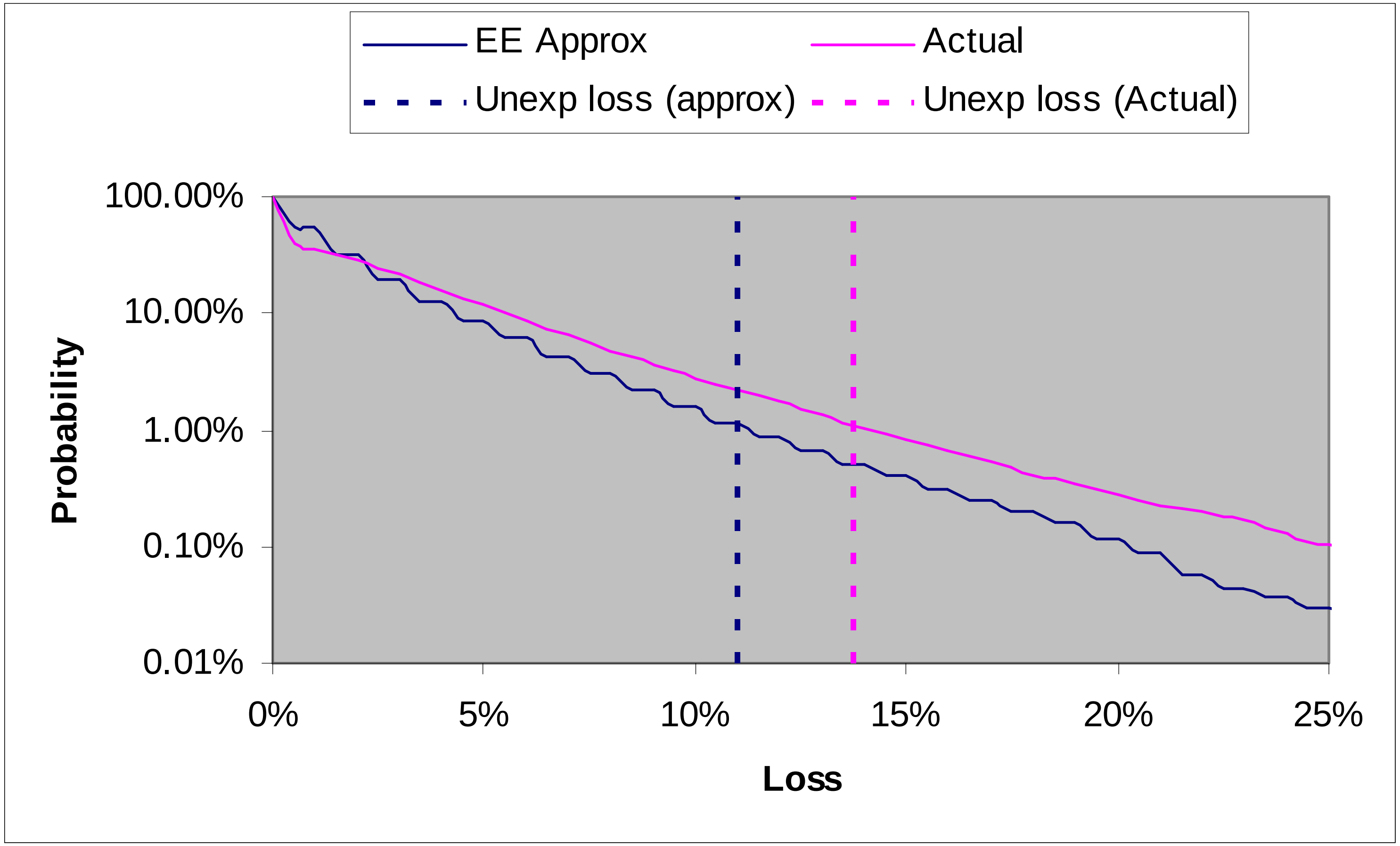
- CVA represents an option on an underlying derivative
  - CVA calculation always harder than pricing the derivative itself
- Need the default probability (and recovery rate) of the counterparty
  - Often market implied probabilities are not known (no CDS market)
- Derivatives are subject to netting agreements
  - Need to price all other trades with this counterparty as well as trade in question
  - All correlations (same asset class, cross-asset class must be known)
- Wrong way risk
  - Linkage between default probability and exposure at default
- Collateral agreements, break clauses etc

CVA – Risk-Neutral or Not?

- Actuarial
  - Consistent with loan book management
  - Insurance company style approach is easier
  - No hedging
- Risk-neutral
  - Consistent with derivatives valuation
  - But trading function for CVA is very difficult to run
  - Hedging is extremely difficult or impossible
- Regulators favour the risk-neutral (mark-to-market) approach
  - But recent problems with hedging in the turbulent Eurozone possibly question this
  - And loans are not treated this way (a derivative is essentially an exotic loan)

CVA and Capital

Alpha and Basel II



Alpha as defined in Basel II

- Basel 2 requires capital to be held against derivatives exposures
- Calculation covers
  - Default risk
  - Credit migration risk (through maturity adjustment factor)
- Alpha adjusts for
  - Exposure volatility
  - Correlation of exposures
  - Size of portfolio (and granularity)

Alpha	**Origin**
1.0	Infinitely large portfolio and independent exposures (theoretical result only)
1.4	Supervisory value
1.2	Supervisory floor when bank uses own model for estimate
1.05 - 1.10	Typical value for large portfolios
> 2.5	Possible value for concentrated portfolios

Regulatory Reaction to the Credit Crisis

- BCBS Committee (Dec 2009)
  - …. where current treatment did not adequately capitalise for risks during the crisis ☺
- Key problems identified
  - Capitalisation of CVA volatility (2/3 of counterparty risk related losses during crisis?)
  - Initial margining (capital to give incentive for adequate initial margin through cycle)
  - Central counterparties not utilised
  - Close-out periods
  - Interconnection of financial institutions
  - Lack of back-testing and stress testing
  - Wrong-way risk

Basel 3 Proposal – CVA "VAR"

- Previous Basel 2 rules account only for default losses (and to some extent credit migration losses)
- Simple capital add-on for CVA risk (bond equivalent)
  - Notional of bond is defined by quantifying future exposure
  - Spread is the one used to calculate CVA (actual or proxy)
  - Maturity of bond is maximum effective maturity of all netting sets for that counterparty
- Risk is then defined as a market risk charge on this bond portfolio
  - VAR type 99% confidence level and 1-year period (may use scaled 10-day)
  - Accounts for hedging using single name CDS and CCDS (or similar instruments) only

The Problems With CVA VAR

- Recent changes
  - Remove the multiplier of 5 (scaling from 10 days to 1 year) ☺
- Only single name hedges (CDS, CCDS) given capital relief
  - Now seemingly will give some relief for index hedges
  - But how? And will this not be encourage procyclicality?
- Methodology
  - Intended to capture in a simple way the credit spread risk within CVA
  - Actually, it is not the optimal way to do this and can lead to non economic results (Rebonato et al.)
- Motivation
  - OTC derivatives are relatively precisely valued, their VAR is much harder to quantify
  - CVA itself is hard to quantify so CVA VAR is surely a major challenge?

DVA

Unilateral CVA in the Old Days

	Credit Rating	Credit spread (bps)
Bank	Aa1/AA+	10-15
Corporate	A3/A-	200-300

- Bank has no default risk
  - Bank charges corporate unilateral CVA
  - If corporate asks for banks default probability to be taken into account, they get laughed at
- No CVA charges in interbank market (collateralised, banks won't default)
- When bank credit quality deteriorates, market becomes gridlocked

Pricing Bilateral Counterparty Risk

- Bilateral CVA considers also an institutions own default (this formula assumes independent of defaults)

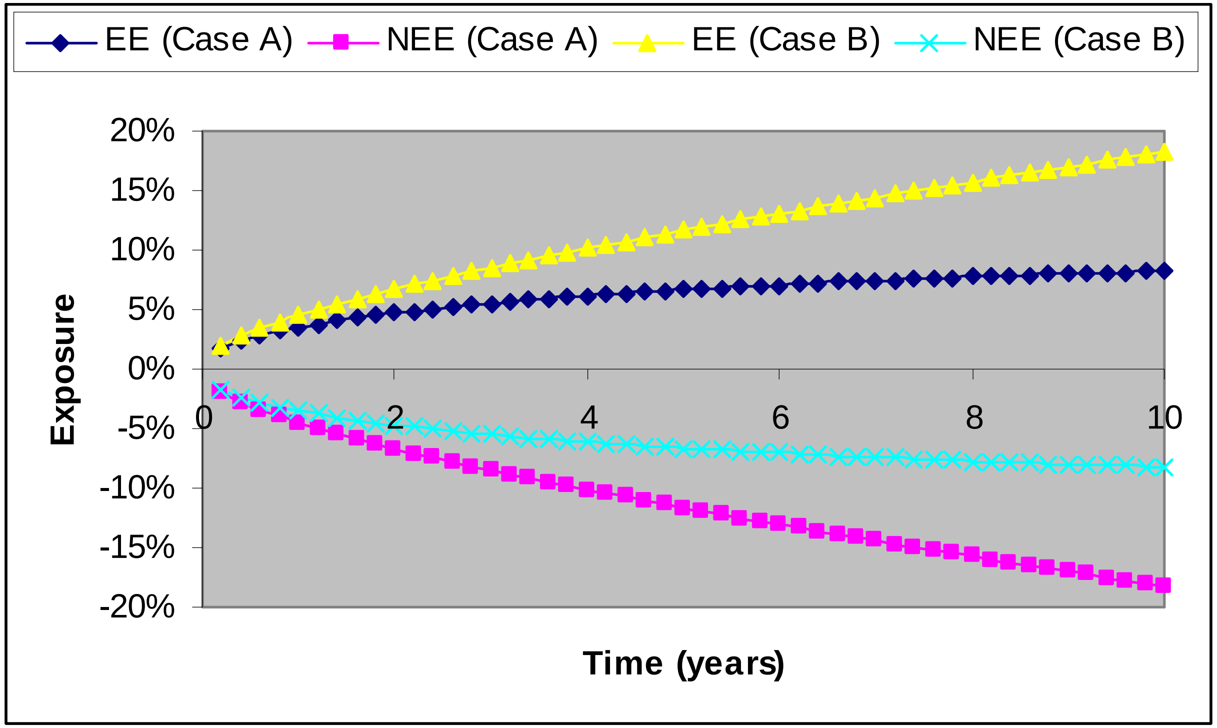
$$BCVA(t) = (1-\delta_C)\int_t^T \underbrace{EE(u)}_{\text{Expected exposure}} \underbrace{[1-PD_I(u)]}_{\text{Probability we haven't yet defaulted}} \underbrace{dPD_C(u)}_{\text{Probability counterparty defaults}} \qquad \text{CVA}$$

$$-(1-\delta_I)\int_t^T \underbrace{NEE(u)}_{\text{Negative expected exposure}} \underbrace{[1-PD_C(u)]}_{\text{Probability \textbf{counterparty} hasn't yet defaulted}} \underbrace{dPD_I(u)}_{\text{Probability \textbf{we} default}} \qquad \text{DVA}$$

Own percentage recovery value

Computing the Bilateral Price

- Bilateral CVA Example
  - Case A : Counterparty 250 bps CDS, Institution 500 bps CDS, EE < NEE
  - Case B : Counterparty 500 bps CDS, Institution 250 bps CDS, EE > NEE



	Case A	Case B
CVA	1.235%	3.480%
BCVA	-1.967%	1.967%

Default Correlation

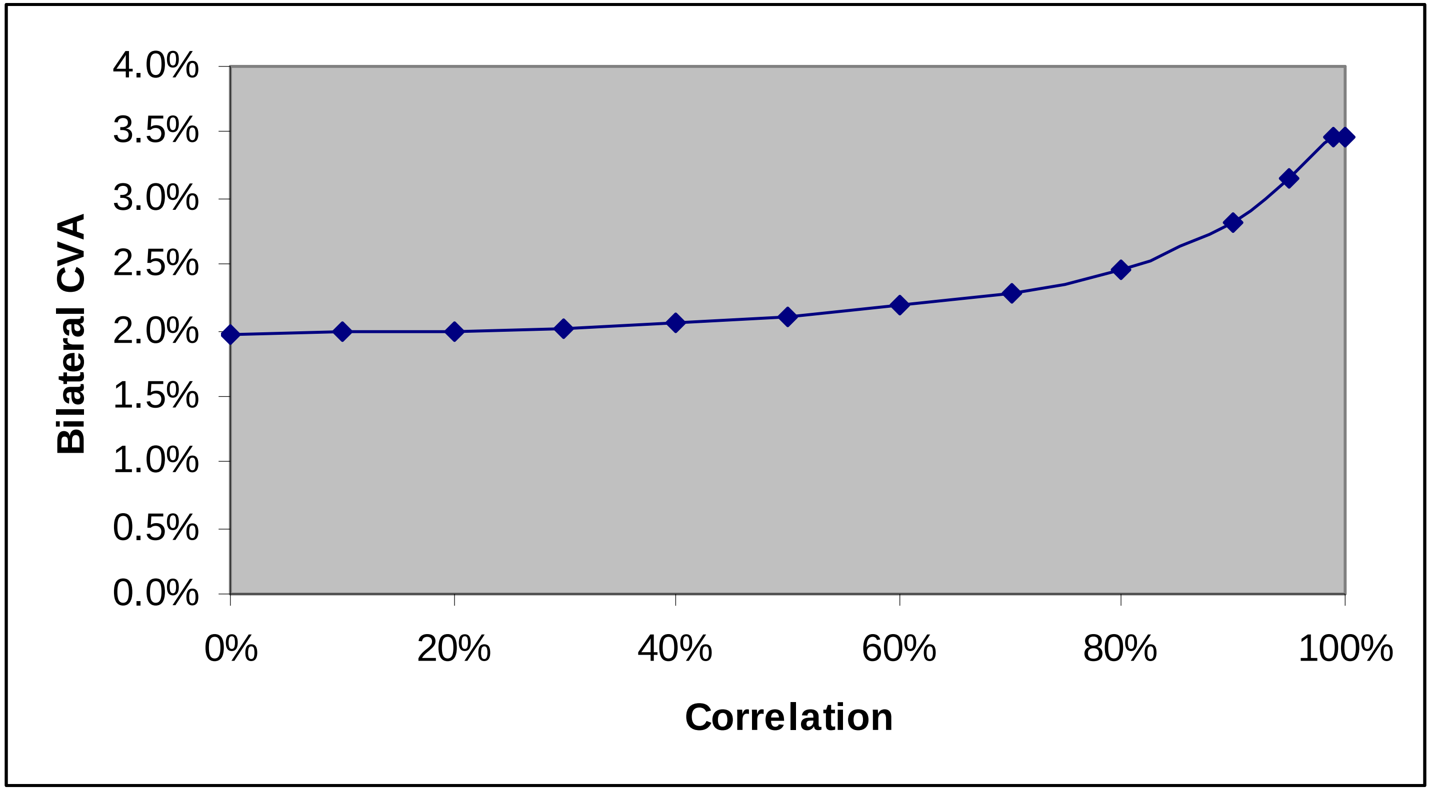
- Gaussian copula approach can be used to give simple tractable correlation between our own default and that of our counterparty
  - Just requires bivariate Gaussian distribution function
  - For example, probability our counterparty defaults in an interval but we don't

$$Q(\tau_C \in [t_{i-1}, t_i], \tau_I > t_i, \tau > t_i) = Q(\tau_C > t_{i-1}, \tau_I > t_i, \tau > t_i) - Q(\tau_C > t_i, \tau_I > t_i, \tau > t_i)$$

$$\approx \begin{bmatrix} \Phi_{2d}\left(\Phi^{-1}\left(Q(\tau_C > t_{i-1})\right), \Phi^{-1}\left(Q(\tau_I > t_i)\right); \rho\right) \\ -\Phi_{2d}\left(\Phi^{-1}\left(Q(\tau_C > t_i)\right), \Phi^{-1}\left(Q(\tau_I > t_i)\right); \rho\right) \end{bmatrix} Q(\tau_C > t_i)$$

Impact of Correlation on BCVA

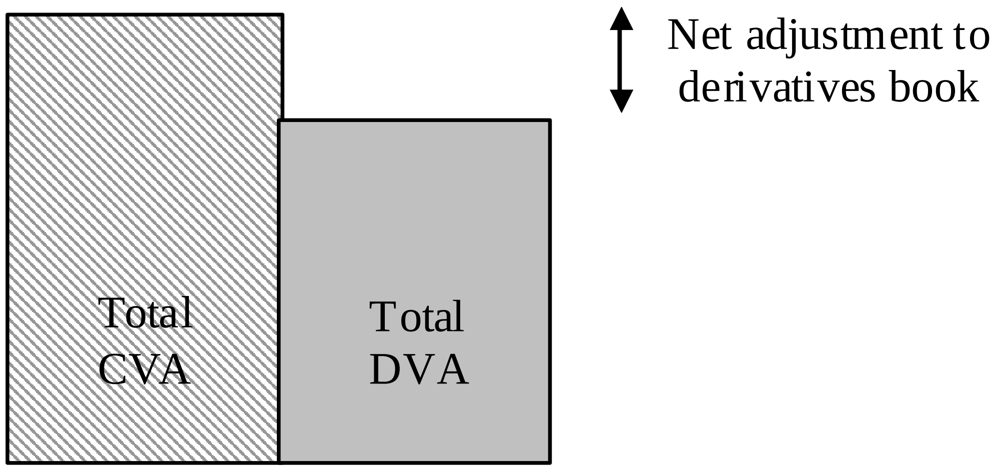
- Case B from previous example
  - Counterparty 500 bps CDS, Institution 250 bps CDS, EE > NEE



Base Case	
CVA	3.480%
BCVA	1.967%

Impact of DVA

$$\text{Bilateral CVA} \approx \underbrace{EPE \times \text{Counterparty spread}}_{\text{CVA}} - \underbrace{ENE \times \text{Institution spread}}_{\text{DVA}}$$



Does Bilateral CVA Make Sense?

- Bilateral CVA has been widely adopted
  - Many banks base CVA on their own default
  - Accountancy rules **<u>require</u>** this (e.g. FAS 157)
- Bilateral CVA has some potentially unpleasant features
  - Total amount of CVA in the market sums to zero
  - Risky value may exceed risk-free value
  - Netting and collateral may increase CVA+DVA
  - Hedging this component is problematic

CUTTING EDGE. CREDIT DERIVATIVES

Being two-faced over counterparty credit risk

*A recent trend in quantifying counterparty credit risk for over-the-counter derivatives has involved taking into account the bilateral nature of the risk so that an institution would consider their counterparty risk to be reduced in line with their own default probability. This can cause a derivatives portfolio with counterparty risk to be more valuable than the equivalent risk-free positions. In this article, **Jon Gregory** discusses the bilateral pricing of counterparty risk and presents a simple approach that accounts for default of both parties. He derives results and then argues that the full implications of pricing bilateral counterparty risk must be carefully considered before it is naively applied for risk quantification purposes*

Counterparty credit risk is the risk that a counterparty in a financial contract will default prior to the expiry of the contract and fail to make future payments. Counterparty risk is taken by each party in an over-the-counter derivatives contract and is present in all asset classes, including interest rates, foreign exchange, equity derivatives, commodities and credit derivatives. Given the recent decline in credit quality and heterogeneous concentration of credit exposure, the high-profile defaults of Enron, Parmalat, Bear Stearns and Lehman Brothers, and writedowns associated with insurance purchased from monoline insurance companies, the topic of counterparty risk management remains ever-important.

A typical financial institution, while making use of risk mitigants such as collateralisation and netting, will still take a significant amount of counterparty risk, which needs to be priced and risk-managed appropriately. Over the past decade, financial institutions have built up their capabilities for handling counterparty risk and active hedging has also become common, largely in the form of buying credit default swap (CDS) protection to mitigate large exposures (or future exposures). Some financial institutions have a dedicated unit that charges a premium to each business line and in return takes on the counterparty risk of each new trade, taking advantage of portfolio-level risk mitigants such as netting and collateralisation. Such units might operate partly on an actuarial basis, utilising the diversification benefits of the exposures, and partly on a risk-neutral basis, hedging key risks such as default and forex volatility.

A typical counterparty risk business line will have significant reserves held against some proportion of expected and unexpected losses, taking into account hedges. The recent significant increases in credit spreads, especially in the financial markets, will have increased such reserves and/or future hedging costs associated with counterparty risk. It is perhaps not surprising that many institutions, notably banks, are increasingly considering the two-sided or bilateral nature when quantifying counterparty risk. A clear advantage of doing this is that it will dampen the impact of credit spread increases by offsetting the associated increase in required reserves. However, it requires an institution to attach economic value to its own default, just as it may expect to make an economic loss when one of its counterparties defaults. While it is true that a corporation does 'gain' from its own default, it might at first glance appear unusual to price this component. In this article, we will make a quantitative analysis of the pricing of counterparty risk and use this to draw conclusions about the validity of bilateral pricing.

Unilateral counterparty risk

The reader is referred to Pykhtin & Zhu (2006) for an excellent overview of measuring counterparty risk. We denote by $V(s, T)$ the value at time s of a derivatives position with a final maturity date of T. The value of the position is known with certainty at the current time $t (< s \leq T)$. We note that the analysis is general in the sense that $V(s, T)$ could indicate the value of a single derivatives position or a portfolio of netted positions[1], and could also incorporate effects such as collateralisation. In the event of default, an institution must consider the following two situations:

- $V(s, T) > 0$. In this case, since the netted trades are in the institution's favour (positive present value), it will close out the position but retrieve only a recovery value, $V(s, T)\delta_C$, with δ_C a percentage recovery fraction.
- $V(s, T) \leq 0$. In this case, since the netted trades are valued against the institution, it is still obliged to settle the outstanding amount (it does not gain from the counterparty defaulting).

- How to monetise bilateral CVA to justify paying for counterparty risk?

How to Realise DVA

- Go bankrupt
  - Usually not a popular choice
- Unwinds or novations
  - An institution may realise a DVA gain if a trade is unwound in the future (e.g. banks unwinding transactions with monolines)
- Hedging
  - DVA much harder to hedge than CVA - cannot sell CDS protection on yourself!
  - Buy back your own debt (not really a dynamic hedge) – do you have the cash?
  - Sell CDS on another counterparty (who is highly correlated with you) – give wrong-way risk to buyer of protection – careful who you choose (Lehman)
- Funding arguments
  - EE represents a funding cost, NEE represents a funding benefit

Hedging Intuition of DVA

- Following Sorenson and Bollier [1994]

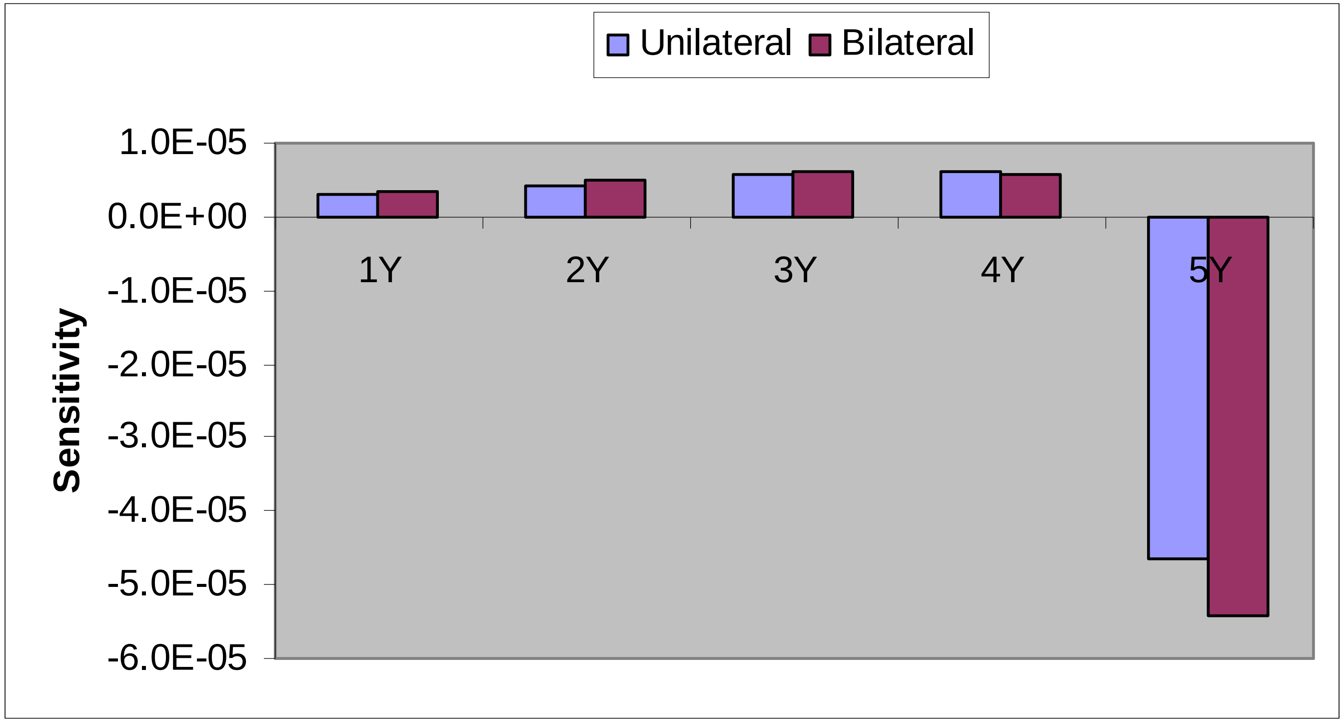
$$CVA_{swap} \approx (1-\delta_C)\sum_{i=1}^{n} S_I(t_{k-1})\left[S_C(t_{k-1}) - S_C(t_k)\right]V_{swaption}(t; t_k, T)$$

$$DVA_{swap} \approx (1-\delta_I)\sum_{i=1}^{n} S_C(t_{k-1})\left[S_I(t_{k-1}) - S_I(t_k)\right]V_{swaption}(t; t_k, T)$$

- Intuition
  - Short a series of swaptions (on reverse swap) with weights given by the forward default probabilities (of counterparty)
  - Long a series of swaptions (on reverse swap) with weights given by the forward default probabilities (of self)
- Hence, using DVA may balance sensitivities

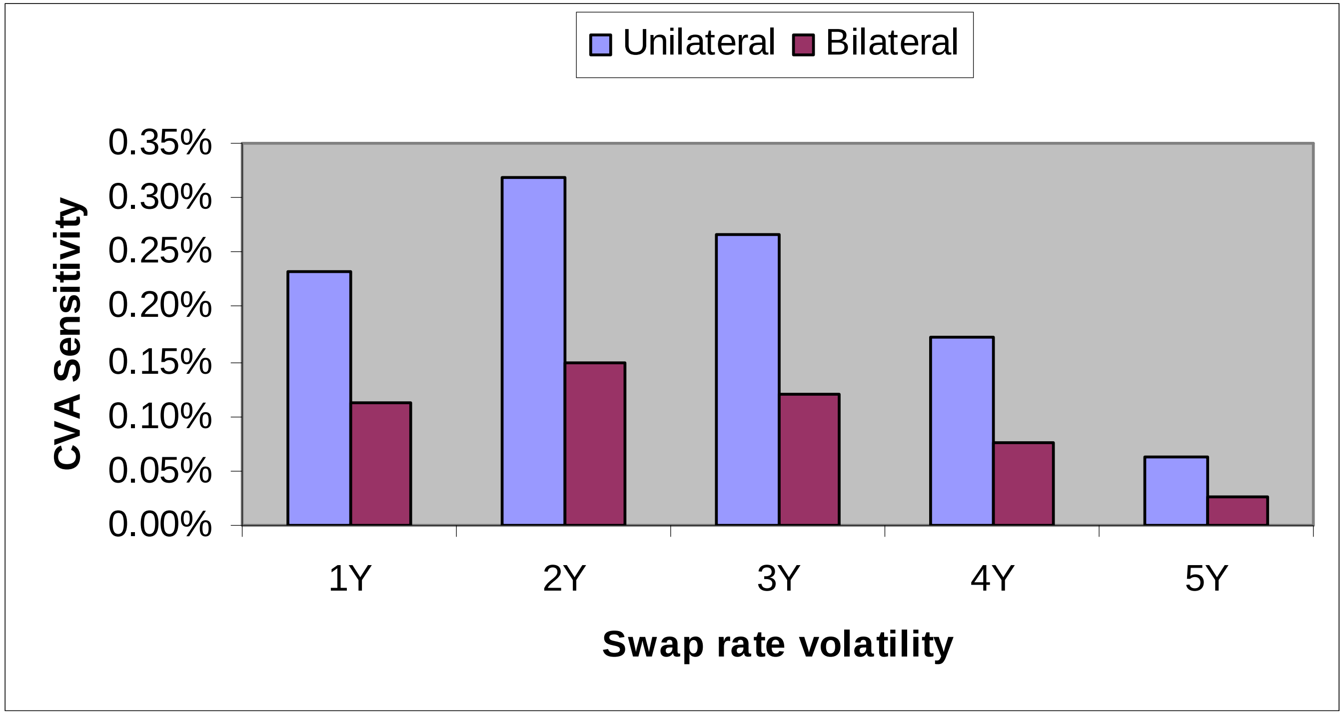
Hedging Using DVA (I)

- Sensitivity to interest rates
  - If CVA increases (for example interest rates go up for a payer swap)
  - Then DVA will decrease
  - Overall sensitivity is increased



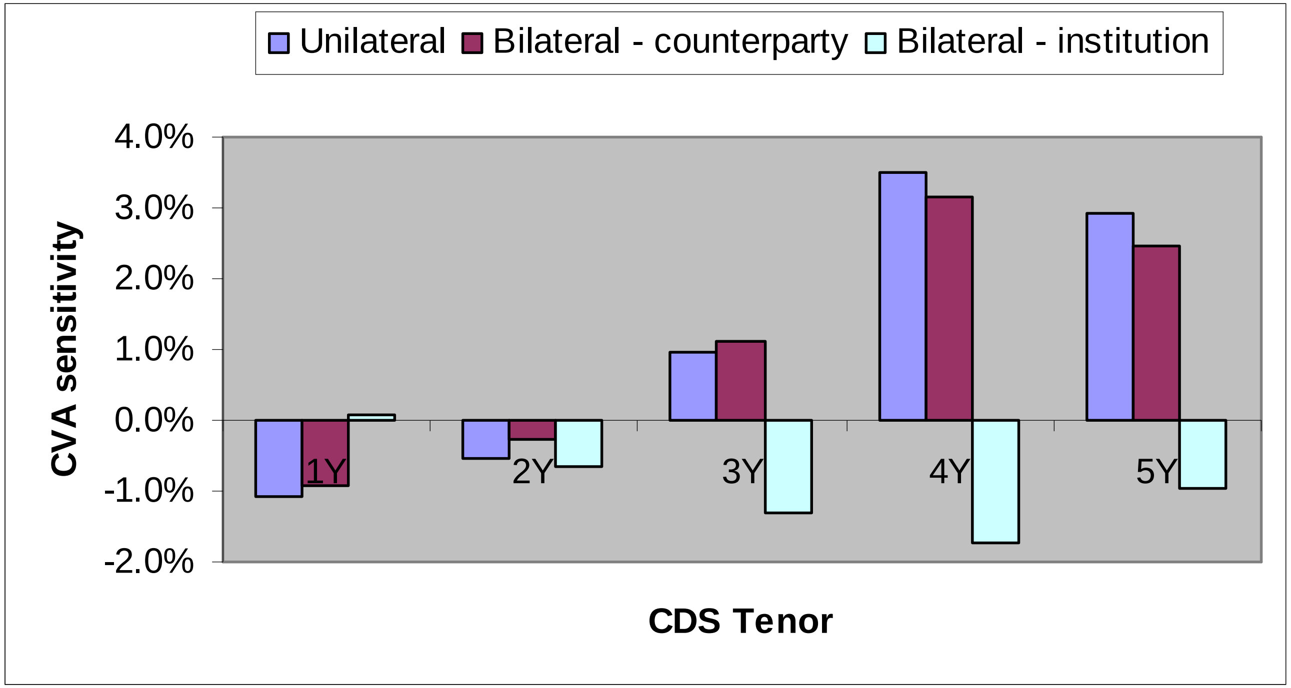
Hedging Using DVA (II)

- Sensitivity to volatility
  - Long and short swaptions will cancel
  - In this case we are half as risky as counterparty (CDS = 250 bps vs 500 bps)
  - Sensitivity is approximately halved



Hedging Using DVA (III)

- Impact of DVA on CDS hedges
  - Buy slightly less protection on counterparty (due to possibility of self defaulting first)
  - Sell protection on oneself
  - Actually made easier by the absence of single name hedges (index beta effect)



DVA and Funding

Funding Costs and CVA / DVA

	Measure	Exposure	Default probability
Default	CVA	EPE	Counterparty credit spread
	DVA	ENE	Own credit spread
Funding	Funding cost	EPE	Own funding spread
	Funding benefit	ENE	Own funding spread

Double counting

Double Counting of Funding

- CVA of a single cashflow

$$CVA = \mathrm{E}\left[e^{-(r+X_I)T} 1_{\tau_C > T}\right]$$

$$= e^{-rT} \times \underbrace{e^{-X_I T}}_{\text{Funding cost}} \times \underbrace{e^{-X_C T}}_{\text{Default risk}}$$

X_I = Funding spread

- DVA

$$DVA = \mathrm{E}\left[e^{-(r+X_I)T} 1_{\tau_I > T}\right]$$

$$= e^{-rT} \times \underbrace{e^{-X_I T}}_{\text{Funding gain}} \times \underbrace{e^{-X_I T}}_{\text{Default risk (own)}} = e^{-rT} \times e^{-2X_I T}$$

Funding and DVA – Some Relevant Papers

- Fries, C., 2010, “Discounting revisited: valuation under funding, counterparty risk and collateralization”
- Morini and Prampolini., 2010, “Risky funding: a unified framework for counterparty and liquidity risk”
- Piterbarg, V., 2010, “Funding beyond discounting: collateral agreements and derivatives pricing”

- We’ll follow the Morini and Prampolini notation but ignore the CDS-bond basis and assume zero recovery rates
- Note we are considering the case of no CSA (collateral)

Funding and DVA – The Key Concept

- For unsecured funding, I pay a funding spread of say X_I
- But I don't pay the funding back if I default τ_I
- Hence, when I pay back the funding of *L* a time Δt later, I pay

$$Le^{r\Delta t}e^{X_I \Delta t}1_{\tau_I > T}$$

- The discounted expectation of this is then

$$Le^{-r\Delta t}e^{r\Delta t}e^{X_I \Delta t}e^{-X_I \Delta t} = L$$

- Funding cost therefore doesn't depends on my credit spread
- This is the accountants view but should it be the quants view?

The Simple Derivative

Binary event

B

Exposure of *L*
Owed money
Funding cost

Δt

Rec from counterparty $Le^{r\Delta t}$
Pay back funding

$1-B$

Exposure of -*L*
Owe money
Funding benefit

Δt

Pay to counterparty $Le^{r\Delta t}$
Release funding

- We can think of this as a simple swap with only 2 possible market scenarios and one time period

Case 1

B

Exposure of *L*

Owed money

Funding cost

Δt

$$\left[\underbrace{-Le^{(r+X_I)\Delta t}}_{\text{Pay back funding}} + \underbrace{Le^{r\Delta t}}_{\text{Receive cashflow}}\right]1_{\tau_A > \Delta t}$$

$1-B$

Exposure of -*L*

Owe money

Funding benefit

Δt

$$\left[\underbrace{+Le^{(r+X_I)\Delta t}}_{\text{Rec back funding}} - \underbrace{Le^{r\Delta t}}_{\text{Pay cashflow}}\right]1_{\tau_A > \Delta t}$$

- This is similar to a contingent swap or clean asset swap (swap cancelled on the basis of a credit event A) with risk-free counterparties

Contingent Swap – Valuation

$$\text{Payoff} = B\left[-Le^{(r+X_I)\Delta t} + Le^{r\Delta t}\right]1_{\tau_A > \Delta t} + (1-B)\left[Le^{(r+X_I)\Delta t} - Le^{r\Delta t}\right]1_{\tau_A > \Delta t}$$

- Price (no wrong way risk)

$$V = E[B]\left[\underbrace{-Le^{X_I \Delta t}}_{\text{Funding cost}} \underbrace{+L}_{\text{Rec}}\right]e^{-X_A \Delta t} + E[1-B]\left[\underbrace{+Le^{X_I \Delta t}}_{\text{Funding benefit}} \underbrace{-L}_{\text{Pay}}\right]e^{-X_A \Delta t}$$

- If $E[B] = q = 1-q$ then $V = 0$ with hedging implication that we need to hedge market risk and buy or sell protection on credit A and consider the need to charge for these dynamic hedging costs
- Mirror trades then $\text{Payoff} = 0$

Case 2

B

Exposure of *L*
Owed money
Funding cost

Δt

$$\underbrace{-Le^{(r+X_I)\Delta t}}_{\text{Pay back funding}}\ \underbrace{+Le^{r\Delta t}1_{\tau_A>\Delta t}}_{\text{Receive}}$$

$1-B$

Exposure of -*L*
Owe money
Funding benefit

Δt

$$\underbrace{+Le^{(r+X_I)\Delta t}}_{\text{Rec back funding}}\ \underbrace{-Le^{r\Delta t}}_{\text{Pay}}$$

- This is similar to a risky swap with counterparty risk where we consider ourselves default free (by the market does not of course)

Unilateral Risky Swap – Valuation

$$\text{Payoff} = B\left[-Le^{(r+X_I)\Delta t} + Le^{r\Delta t}1_{\tau_A > \Delta t}\right] + (1-B)\left[+Le^{(r+X_I)\Delta t} - Le^{r\Delta t}\right]$$

$$V = E[B]\left[\underbrace{-Le^{-X_I\Delta t}}_{\text{Funding cost}} + \underbrace{Le^{-X_A\Delta t}}_{\text{CVA component}}\right] + E[1-B]\left[\underbrace{+Le^{-X_I\Delta t}}_{\text{Funding benefit}} - L\right]$$

- Mirror trades with two different counterparties A and B

$$V_{AB} = E[B]\left[-Le^{-X_I\Delta t} + Le^{-X_A\Delta t} + Le^{-X_I\Delta t} - L\right]$$
$$+ E[1-B]\left[+Le^{-X_I\Delta t} - L - Le^{-X_I\Delta t} + Le^{-X_B\Delta t}\right]$$

- Funding cancels, the trade has negative value for $X_A, X_B > 0$

$$V_{AB} = E[B].L\left[e^{-X_A\Delta t} - 1\right] + E[1-B]L\left[e^{-X_B\Delta t} - 1\right]$$

Case 3

B

Exposure of *L*

Owed money

Funding cost

Δt

$$\underbrace{-Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t}}_{\text{Pay back funding}}\ \underbrace{+Le^{r\Delta t}1_{\tau_A>\Delta t}}_{\text{Receive from A}}$$

$1-B$

Exposure of -*L*

Owe money

Funding benefit

Δt

$$\underbrace{+Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t}}_{\text{Rec back funding}}\ \underbrace{-Le^{r\Delta t}1_{\tau_I>\Delta t}}_{\text{Pay to A}}$$

- This is similar to a risky swap where both counterparties may default

Bilateral Risky Swap – Valuation (I)

$$\text{Payoff} = B\left[-Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} + Le^{r\Delta t}1_{\tau_A>\Delta t}\right] + (1-B)\left[Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} - Le^{r\Delta t}1_{\tau_I>\Delta t}\right]$$

$$V = E[B]\Big[\underbrace{-L}_{\text{Funding cost}} + \underbrace{Le^{-X_A\Delta t}}_{\text{CVA}}\Big] + E[1-B]\Big[\underbrace{+L}_{\text{Funding benefit}} - \underbrace{Le^{-X_I\Delta t}}_{\text{DVA}}\Big]$$

- If $E\left[B\right] = q = 1-q$ $\quad V = Le^{-r\Delta t}\left[e^{-X_A\Delta t} - e^{-X_I\Delta t}\right]/2$
- Funding cancels in expectation (but still have funding risk)
- Hedging implications
  - Hedge market risk
  - Buy protection on A, sell protection on ourselves
  - Consider hedging costs even when $X_A = X_I$

Bilateral Risky Swap – Valuation (II)

- Mirror trades with two different counterparties A and B

$$\text{Payoff} = B\left[-Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} + Le^{r\Delta t}1_{\tau_A>\Delta t} + Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} - Le^{r\Delta t}1_{\tau_I>\Delta t}\right]$$
$$+(1-B)\left[Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} - Le^{r\Delta t}1_{\tau_I>\Delta t} - Le^{(r+X_I)\Delta t}1_{\tau_I>\Delta t} + Le^{r\Delta t}1_{\tau_B>\Delta t}\right]$$

- Funding cancels

$$\text{Payoff} = Be^{r\Delta t}\left[+L1_{\tau_A>\Delta t} - L1_{\tau_I>\Delta t}\right] + (1-B)e^{r\Delta t}\left[-L1_{\tau_I>\Delta t} + L1_{\tau_B>\Delta t}\right]$$

- Valuation

$$V_{AB} = E[B]L\left(e^{-X_A\Delta t} - e^{-X_I\Delta t}\right) + E[1-B]L\left(e^{-X_B\Delta t} - e^{-X_I\Delta t}\right)$$

- Same comments as before on hedging
- In this case **DVA is clearly not a funding benefit**

Should you use DVA?

- On the one hand, firms need to use DVA
  - Reduces CVA charges
  - Likely that both counterparties to a trade will agree a price
  - Reduces volatility of CVA desk's book and hedging costs
- On the other hand
  - **Cannot be treated as a funding benefit**
  - Requires a firm to see their future default as a good thing and try and monetise it
  - Does not encourage good practices for a CVA desk
  - For example, a firm going to default will need to sell more and more CDS protection (and more and more volatility)