Being Two-Faced Over Counterparty Credit Risk?

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A recent trend in pricing counterparty credit risk for OTC derivatives has involved taking into account the bilateral nature of the risk so that an institution would reduce counterparty risk in line with their own default probability. Done to an extreme, this practice has worrying implications such as causing a derivatives portfolio with counterparty risk to be more valuable than the equivalent risk-free positions. In this paper, we argue that to apply naïve bilateral pricing of counterparty risk is rather dangerous and we present a simple realistic approach for pricing and hedging counterparty risk in OTC derivatives accounting for default of both parties.

Counterparty credit risk is the risk that a counterparty in a financial contract will default prior to the expiration of the contract and fail to make future payments. Counterparty risk is taken by each party in an over-the-counter (OTC) derivatives contract and is present in all asset classes, including interest-rates, foreign-exchange, equity derivatives, commodities and credit derivatives. Given the recent decline in credit quality and heterogeneous concentration of credit exposure, together with high profiles defaults of Enron, Parmalat and Bear Stearns, the topic of counterparty risk management remains an important one.

A typical financial institution, whilst making use of risk mitigants such as collateralisation and netting, will still take a significant amount of counterparty risk which needs to be priced and risk managed appropriately. Over the last decade, financial institutions have built up their capabilities for handling counterparty risk and active hedging has also become common, largely in the form of buying credit default swap (CDS) protection to mitigate large exposures (or future exposures). Most banks have a dedicated counterparty risk management unit which will charge a premium to each business line to bear the counterparty risk of a new trade, taking advantage of portfolio level risk mitigants such as netting and collateralisation. Such might operate partly on an actuarial basis, making use of the diversification benefits of the exposures, and partly on a risk-neutral basis hedging key risks such as default and FX volatility.

A typical counterparty risk business line will have significant reserves held against some proportion of expected and unexpected losses, taking into account hedges. The

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recent significant increases in credit spreads, especially in the financial space will have increased the reserves and/or future hedging costs associated with counterparty risk. It is perhaps not surprising then that many institutions, notably banks, are increasingly considering the two-sided or bilateral nature when quantifying counterparty risk. A clear advantage of doing this is the it will dampen the impact of significant credit spread increases by offsetting the associated increase in required reserves. However, it means that they are attaching economic value to the fact that, just as they may make a loss when a counterparty defaults, they would gain if they themselves default. Whilst it is strictly true that they do indeed gain from their own default, it might at first glance appear unusual to price this component. In this paper, we will make a quantitative analysis of the pricing of counterparty risk and attempt to draw conclusions about the validity of bilateral pricing.

Unilateral Counterparty Risk

The reader is referred to the article of Pykhtin and Zhu [2006] for an excellent overview of measuring counterparty credit risk. We denote the current time by t and use $V(s,T)$ to describe the unknown value of a derivatives position with a final maturity date of T with $t \leq s \leq T$. We note that the analysis is general in the sense that $V(s,T)$ could indicate the value of a single derivatives position or a portfolio and could also incorporate effects such as netting and collateralisation. In the event of default then an institution must consider the following two situations.

$V(s,T) > 0$ In this case the trade is in the institutions favour (positive present value) and they will close out the position but retrieve only a recovery value, $V(s,T)\delta_C$ with δ_C a percentage recovery fraction.

$V(s,T) \leq 0$ In this case the trade is valued against the institution and there is no additional loss although they are still obliged to settle the outstanding amount (they do not gain from the counterparty defaulting).

We can therefore write the payoff in default as $\delta_C V(\tau_C,T)^+ + V(\tau_C,T)^-$ where τ_C is the default time of the counterparty[2]. The risky value of a trade or portfolio of trades where the counterparty may default at some time in the future is then: -

$$\tilde{V}(t,T) = E\left[1_{\tau_C > T} V(t,T) + 1_{\tau_C \leq T}\left(V(t,\tau_C) + \delta_C V(\tau_C,T)^+ + V(\tau_C,T)^-\right)\right] \qquad (1)$$

The first term in the expectation is simply the risk-free value conditional upon no default before the final maturity. The second component $1_{\tau_C \leq T} V(t,\tau_C)$ corresponds to the cashflows paid before the default time. The final components can be identified as the default payoff as described above.

Re-arranging the above equation, we obtain: -

[2] We use the notation $x^+ = \max(x,0)$ and $x^- = \min(x,0)$.

$$
\begin{aligned}
&\tilde{V}(t,T) = E\left[1_{\tau_C > T} V(t,T) + 1_{\tau_C \le T}\left(V(t,\tau_C) + \delta_C V(\tau_C,T)^+ + V(\tau_C,T) - V(\tau_C,T)^+\right)\right] \\
&= E\left[1_{\tau_C > T} V(t,T) + 1_{\tau_C \le T} V(t,T) + 1_{\tau_C \le T}\left(\delta_C V(\tau_C,T)^+ - V(\tau_C,T)^+\right)\right] \\
&= V(t,T) - E\left[1_{\tau_C \le T}(1-\delta_C) V(\tau_C,T)^+\right]
\end{aligned}
\tag{2}
$$

This allows us to express the risky value as the risk-free value less an additional component. This component is often referred to (for example see Pykhtin and Zhu [2006]) as the credit value adjustment (CVA). As first discussed by Sorensen and Bollier [1994], an analogy is often made that the counterparty is long a series of swaptions. Let us denote the standard CVA in this unilateral case as: -

$$
CVA_{unilateral} = E\left[1_{\tau_C \le T}(1-\delta_C) V(\tau_C,T)^+\right] \tag{3}
$$

We might compute the expectation under the risk-neutral or the real probability measure, in the latter case using historical analysis of price data rather than market implied parameters. Traditionally the real measure is often used in risk management applications involving modelling future events such as exposures. However, since the default component of the CVA is likely to be hedged, the risk-neutral measure might be more appropriate. Since most counterparty risk books may hedge only the major risks and are therefore part risk-neutral, part actuarial we would argue that the measure to use in equation (3) becomes a rather subtle question. Having noted this point, since this paper is concerned with pricing then we will use the risk-neutral measure and will argue that this is precisely where the objection to bilateral pricing originates.

Bilateral Counterparty Risk

A possible over-simplification of the unilateral treatment is that it neglects the fact that the institution may default before their counterparty, in which cases the latter default would be irrelevant to them. Furthermore, the institution could be argued to actually make a gain in their own default since they will pay the counterparty only a fraction of the value of the contract. The payoff to the institution in their own default is $\delta_A V(\tau_I,T)^- + V(\tau_I,T)^+$ with τ_I and δ_I representing their own default time and associated recovery percentage respectively.

Denoting by $\tau^1 = \min(\tau_C,\tau_I)$ the "first-to-default" time of both the institution and counterparty and assuming that simultaneous defaults are not possible, then the valuation equation becomes: -

$$
\begin{aligned}
\tilde{V}(t,T) &= E^Q\left[1_{\tau^1>T}V(t,T)+1_{\tau^1\le T}\begin{pmatrix} V(t,\tau^1)+ \\ 1_{\tau^1=\tau_C}\left(\delta_C V(\tau^1,T)^+ + V(\tau^1,T)^-\right)+ \\ 1_{\tau^1=\tau_I}\left(\delta_A V(\tau^1,T)^- + V(\tau^1,T)^+\right) \end{pmatrix}\right] \\
&= V(t,T) - E^Q\left[1_{\tau^1\le T}\left(1_{\tau^1=\tau_C}(1-\delta_C)V(\tau^1,T)^+ + 1_{\tau^1=\tau_A}(1-\delta_A)V(\tau^1,T)^-\right)\right],
\end{aligned} \tag{4}
$$

We can identify the first component in equation (4) as being the same adjustment as before conditioned on no default of the institution. The additional term corresponds to the “gain” made by the institution in the event of their default (conditional on no previous counterparty default). We note that the asymmetry of the future distribution of $V(\tau^1,T)$ (for example, due to forward rates being far from spot rates) should be important in determining the relative value of the two terms above in addition to the relative default probabilities.

An obvious implication of the bilateral formula is that the overall CVA may be negative, i.e. actually increase the overall value of the derivatives positions. Another worrying implication of the above symmetry is that the overall amount of counterparty risk in the market would be zero[3]. A practical objection to the above formula would be that, whilst the risk-neutral default component of the unilateral CVA can be hedged by buying CDS protection on the counterparty, the additional term in the bilateral formula would require an institution to sell CDS protection on themselves. Whilst it is not directly possible for an institution to short its own debt, the argument could be made that they could hedge by selling protection on a highly correlated credit; for example banks might sell protection on (a basket) of other banks. Such a hedge would cause a bank to lose money when, in line with their presumed wish to outperform their competitors in equity returns, their credit spread tightens relative to the basket.

In order to be able to study the above idea, we extend the CVA formula to allow for a simultaneous default of both parties and denote the time of this event as τ. The overall idea of the approach is that an institution should achieve a reduction in their CVA according to the hedgeable components only[4] (this could be thought of in terms of a beta or as systemic over idiosyncratic risk). In the presence of joint default the valuation formula becomes: -

[3] This assumes that all parties have the same pricing measure in which case the two sides to a trade or netted portfolio of trades will always have equal and opposite CVAs.

[4] An institution selling protection on a correlated basket of names will having a gain/loss when their own spread widens/tightens with respect to the basket.

$$\tilde{V}(t,T) = E^Q\left[1_{\tau^1>T}V(t,T) + 1_{\tau^1\le T}\begin{pmatrix} V(t,\tau^1) + \\ 1_{\tau^1=\tau_C}\left(\delta_C V(\tau^1,T)^+ + V(\tau^1,T)^-\right) + \\ 1_{\tau^1=\tau_A}\left(\delta_I V(\tau^1,T)^- + V(\tau^1,T)^+\right) + \\ 1_{\tau^1=\tau}\left(\delta_C V(\tau^1,T)^+ + \delta_I V(\tau^1,T)^-\right) \end{pmatrix}\right] \tag{5}$$

$$= V(t,T) - E^Q\left[1_{\tau^1\le T}\begin{pmatrix} 1_{\tau^1=\tau_C}(1-\delta_C)V(\tau^1,T)^+ + \\ 1_{\tau^1=\tau_A}(1-\delta_I)V(\tau^1,T)^- \\ 1_{\tau^1=\tau}\left(V(\tau^1,T) - \delta_C V(\tau^1,T)^+ - \delta_I V(\tau^1,T)^-\right) \end{pmatrix}\right] = V(t,T) - CVA_{\text{bilateral}}$$

with $\tau^1 = \min(\tau_C, \tau_I, \tau)$. The final term corresponds to the fact that in the event of joint default, the value of the derivatives position is essentially cancelled, with a recovery value paid to whichever party is owed money. It can be shown that due to the size of this term that this bilateral term above will always be between the unilateral adjustment and the previous bilateral adjustment with no joint defaults[5].

Example

The above formulae are rather general so in order to get some intuition on the problem we now apply a simple model. We first assume that the probabilities of default are determined by: -

$$\begin{aligned} &Q(\tau_C > s) = \exp\left[-(\lambda_C - \lambda)s\right], && \lambda_C \le \lambda \\ &Q(\tau_I > s) = \exp\left[-(\lambda_I - \lambda)s\right], && \lambda_I \le \lambda \\ &Q(\tau > s) = \exp\left[-\lambda s\right], && \text{(6a, 6b, 6c)} \end{aligned}$$

where λ_C, λ_I and λ are deterministic default intensities which could readily be made time dependent or, in a more complex approach, stochastic. The joint default probability λ could be calculated from the prices n^{th} to default baskets or (under the assumption that this will be systemic event) senior tranches of a relevant credit index. Perhaps more practically, λ could be determined empirically via the proportion (beta) of the credit spread of the institution that is hedgeable via CDS or bonds of correlated names. After this then λ_C and λ_I can be calibrated to the credit default swap (CDS) spreads of the counterparty and institution respectively. Since the hazard rates will normally be calibrated to CDS quotes and the recovery rate derivatives under standard ISDA documentation is pari passu with senior debt[6] then we do not expect a considerable impact from differing recovery assumptions.

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[5] This follows from $(1-\delta_I)V(\tau,T)^- \ge V(\tau,T) - \delta_C V(\tau,T)^+ - \delta_A V(\tau,T)^- \ge (1-\delta_c)V(\tau,T)^C$.

[6] We note that there is some additional complexity regarding this point. Firstly, since CDS protection buyers must buy bonds to deliver then a "delivery squeeze" can occur if there is more CDS notional in the market than outstanding deliverable bonds. In this case, the bond price can be bid up and suppress the value of the CDS hedging instrument. This has been seen in many recent defaults such as Parmalat [2003] Delphi [2005] and for many counterparties the amount of CDS traded is indeed larger than

We will make the common assumptions that the default times and value of the derivatives portfolio are independent. This is a rather common assumption in the case that there is not obvious “wrong-way risk” (which clearly exists in credit default swaps and certain other cases). As noted before, the approach described here could be combined with a “wrong-way risk” approach such as Cherubini and Luciano [2002]. Under the independence assumption we obtain: -

$$
\begin{aligned}
&CVA_{bilateral} = \\
&Q(\tau_C \le T, \tau_C < \tau_A \wedge \tau) E^Q\left[(1-\delta_C)V(\tau_C,T)^+\right] + \\
&Q(\tau_I \le T, \tau_I < \tau_I \wedge \tau) E^Q\left[(1-\delta_I)V(\tau_I,T)^-\right] + \\
&Q(\tau \le T, \tau < \tau_C \wedge \tau_I) E^Q\left[V(\tau,T) - \delta_C V(\tau,T)^+ - \delta_I V(\tau,T)^-\right]
\end{aligned}
\tag{7}
$$

The most straightforward way to compute the above integral is by discretisation over a suitable time grid $[t_0 = t, t_1, ...t_{m-1}, t_m = T]$: -

$$
\begin{aligned}
&CVA_{bilateral} \approx \\
&\sum_{i=1}^{m} Q(\tau_C \in [t_{i-1}, t_i], \tau_I > t_i, \tau_C > t_i) E\left[(1-\delta_C)V(\tau_C,T)^+\right] + \\
&\sum_{i=1}^{m} Q(\tau_I \in [t_{i-1}, t_i], \tau_C > t_i, \tau_I > t_i) E\left[(1-\delta_I)V(\tau_I,T)^-\right] + \\
&\sum_{i=1}^{m} Q(\tau^1 \in [t_{i-1}, t_i], \tau_C > t_i, \tau_I > t_i) E\left[V(\tau^1,T) - \delta_C V(\tau^1,T)^+ - \delta_A V(\tau^1,T)^-\right]
\end{aligned}
\tag{8}
$$

We assume that the counterparty and institution default probabilities are correlated according to a Gaussian copula as is standard in structured credit derivatives pricing. The correlation parameter is denoted by ρ. Following the Gaussian correlation assumption between τ_C and τ_I and the independence of τ, the above probabilities can be readily computed, for example: -

$$
\begin{aligned}
&Q(\tau_C \in [t_{i-1}, t_i], \tau_I > t_i, \tau > t_i) = Q(\tau_C > t_{i-1}, \tau_I > t_i, \tau > t_i) - Q(\tau_C > t_i, \tau_I > t_i, \tau > t_i) \\
&\approx \begin{bmatrix} \Phi_{2d}\left(\Phi^{-1}\left(Q(\tau_C > t_{i-1})\right), \Phi^{-1}\left(Q(\tau_I > t_i)\right); \rho\right) \\ -\Phi_{2d}\left(\Phi^{-1}\left(Q(\tau_C > t_i)\right), \Phi^{-1}\left(Q(\tau_I > t_i)\right); \rho\right) \end{bmatrix} Q(\tau_C > t_i)
\end{aligned}
\tag{9}
$$

Where $\Phi(.)$ and $\Phi_{2d}(.)$ represent the univariate and bivariate cumulative normal distribution functions.

We finally use the simple representation $V(s,T) = \mu(s-t) + \sigma\sqrt{s-t}Z$ where μ and σ are drift and volatility parameters respectively and Z is a random variable drawn

available pool of bonds. We also note that whilst CDS are settled shortly after default, derivatives claims go through a workout process which can last years and the final recovery achieved may be rather different.

from a standard normal distribution function[7]. The simple assumptions above allow us to calculate the final required quantities as: -

$$E\left[V(s,T)^{\pm}\right] = \mu s N(\pm \mu s/\sigma) + \sigma\varphi(\pm \mu s/\sigma) \qquad (10)$$

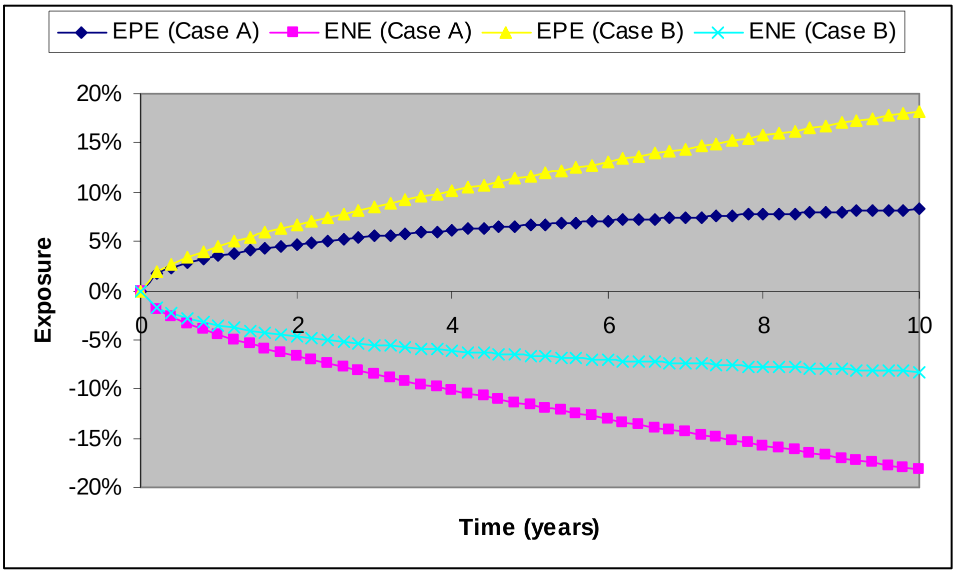
where $\varphi(.)$ represents the normal distribution function. These components are typically known as the expected positive exposure (EPE) and the expected negative exposure (ENE). We are not considering interest rates which under the independence assumptions simply amount to multiplicative components via discount factors.

Let us assume that $\delta_C = \delta_I = 40\%$ and define two parameters sets[8]: -

Case A : $\mu = -1\%$, $\sigma = 10\%$, $\lambda_C = 2\%$, $\lambda_I = 4\%$,
Case B : $\mu = +1\%$, $\sigma = 10\%$, $\lambda_C = 4\%$, $\lambda_I = 2\%$.

The (symmetric) exposure for profiles (EPE and ENE) are shown in Figure 1.

Figure 1. Expected exposure profiles for Case A and Case B with $\mu = -1\%$, $\sigma = 10\%$ and $\mu = +1\%$, $\sigma = 10\%$ respectively.



We will consider three distinct CVA measures outlined below: -

Unilateral : this is the standard unilateral formula given in equation (3).

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[7] For simplicity and since this is a simple example, we ignore the usual risk-neutral restrictions For single cash-flow products, such as FX forwards, or products with a final large cashflow, such as the exchange of principal in a cross-currency swap, the maximum exposure occurs at the maturity of the transaction and this formula proves a good proxy for the typical exposure. Products with multiple cashflows, such as interest-rate swaps typically have a peak exposure between one half and one third of the maturity. We note that the exposure of the same instrument may vary also significantly due to market conditions such as the shape of yield curves. We have confirmed that the qualitative conclusions do not depend on the precise exposure profile chosen.

[8] The constant intensities of default are approximately related to CDS premia via $\lambda(1-\delta)$.

Adjusted Unilateral : this is the unilateral adjustment but taking into account the default probability of the institution, i.e. this is the first term in equation (8) with no joint default probability, $\lambda = 0$.

Bilateral : The bilateral CVA given by equation (8).

Initially we assume zero correlation and zero joint default probability, $\rho = \lambda = 0$ and show the three CVA values in Table 1.

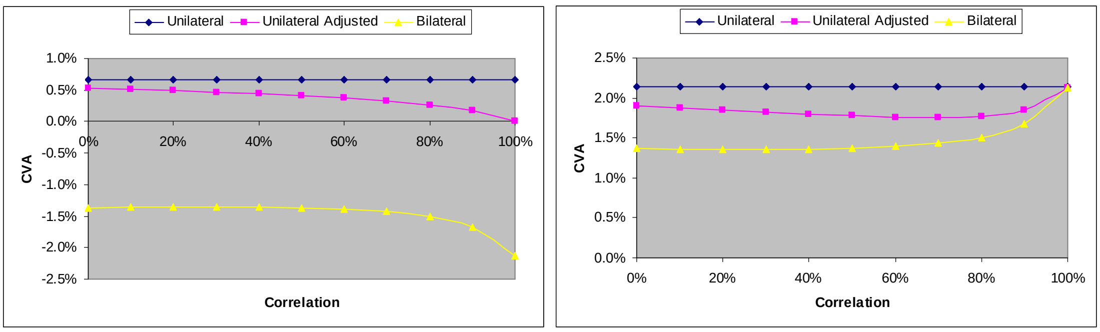
Table 1. Unilateral and bilateral CVA values for case A and case B under the assumption of independence.

	Case A	Case B
Unilateral	0.619%	2.106%
Unilateral Adjusted	0.521%	1.923%
Bilateral	-1.409%	1.397%

Case A represents a situation where the bilateral CVA is negative due to the institution's higher default probability and the high chance that they will owe money on the contract (negative exposure due to $\mu = -1\%$). Case B is the opposite case and, since the counterparty is more risky than the institution, the bilateral CVA is reduced by only around one third compared to the unilateral case. We see that since Case A and Case B represent equal and opposite scenarios for each party, the sum of the bilateral adjustments is zero.

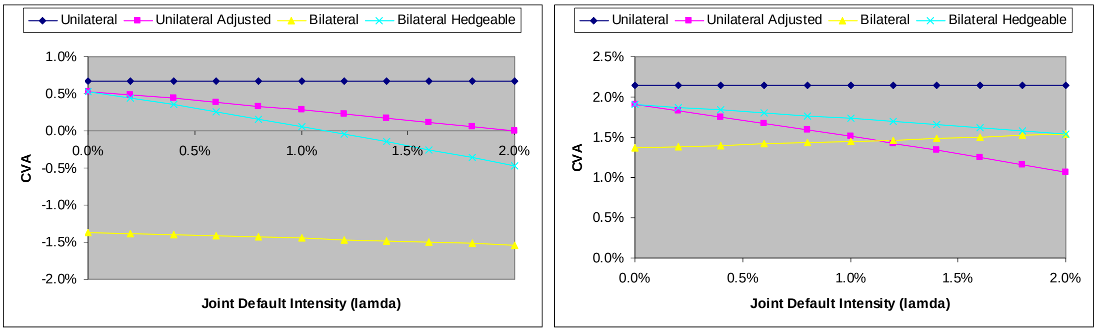
Now let us consider the impact of correlation on the CVA. As shown in Figure 2, correlation can have a reasonably significant impact on both the unilateral and bilateral values. As correlation increases, we approach comonotonicity where the more risky credit is sure to default first. This means that in case A, the unilateral adjusted CVA goes to zero (the institution is sure to default first) whilst in case B it converges to the pure unilateral value (the counterparty is sure to default first).

Figure 2. CVA as a function of correlation between counterparty and institution default for case A (left) and case B (right).



Let us now consider the impact of joint default. We show the three CVA components versus the joint default intensity, $\lambda \leq \min(\lambda_C, \lambda_I)$, together with the "bilateral hedgeable" component which assumes that the second term in equation (7) is zero on account an institution being unable to short their own debt[9]. This means that the CVA is reduced by only the joint default component and not the idiosyncratic default of the institution. In the case of zero joint default then the bilateral hedgeable is the same as the unilateral adjusted CVA.

Figure 3. CVA as a function of the systemic spread intensity with zero correlation.



We see that joint default plays a similar role to correlation and does not have a significant impact on the bilateral CVA. However, from looking at the bilateral hedgeable component, we see that it is quite possible that a substantial portion of the bilateral benefit comes from a component that cannot be moneterised. With no joint default probability then the CVA is simply the unilateral adjusted value and we must assume a significant joint default probability to reduce the CVA. However, we must also have a way to hedge this joint default component.

Conclusion

Appropriate pricing and risk management of counterparty risk is a key area for financial institutions and controlling the level of reserves and cost of hedging is critical in turbulent times. However, realistic pricing and management of risk should always be the key objective. In this paper we have argued that, whilst there should some possibility to reduce counterparty risk charges by the default probability of an institution, a full reduction taking into account bilateral risk is inappropriate. The argument we have made is that an institution should be able to reduce counterparty risk in line with the systemic component of their credit spread which, unlike the idiosyncratic component, is hedgeable. Using a model which represents a simple extension of most counterparty risk pricing approaches, we have illustrated some pricing behaviour. Such ideas can readily be incorporated into most counterparty risk pricing and management functions to attempt to have a reasonable treatment of the bilateral nature of this risk.

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[9] Even if it is argued that the institution does benefit from their own default then since this cannot be hedged then this component should not be priced under the risk neutral measure.

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