Closing out the DVA debate

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When considering counterparty credit risk, it has become increasingly common in recent years for institutions to consider bilateral CVA which includes debt value adjustment (DVA) linked to their own default probability. However, the use of DVA is contentious as it is not obvious that an institution can monetise this "profit" prior to actually defaulting. Furthermore, the use of DVA also seems to require an understanding of the dependence between the default probability of the institution and their counterparty and introduces ambiguity over the correct choice of "closeout" assumptions. In this article, we analyse the complex interaction between CVA, DVA and closeout assumptions and consider if the relatively simple formulas used for bilateral CVA in the industry are reasonable.

1. Introduction

Institutions often consider their own default in the valuation of liabilities. This can be included by pricing of counterparty risk bilaterally, including what is often known as the DVA (debt value adjustment) component. DVA is a double edged sword. On the one hand, it creates a symmetric world where counterparties can readily agree on pricing. On the other hand, the nature of DVA creates some potentially unpleasant effects, such as institutions booking profits arising from their own declining credit quality. The controversy over DVA can be seen when comparing accountancy standards and capital rules. Whilst accounting rules (IFRS 13, FASB 157) require DVA, the Basel III framework does not allow any DVA relief in capital calculations[2].

The debate over DVA usage centres on whether or not institutions can monetise their own default[3]. Ways that institutions attempt to do this include selling CDS protection on similar counterparties[4], buying back own debt and unwinding trades (e.g. see Gregory [2009], Burgard and Kjaer [2011]). Whilst not completely impossible, such techniques are often seen as dubious and only leading to unintended consequences such as the creation of systemic risk. Another possible way to realise DVA is when closing out trades in the event of the default of the counterparty. In such a case, material economic factors, such as DVA can be incorporated into the closeout amount[5] ("risky closeout"). Note that an institution will suffer the reverse experience if it defaults, since counterparties can attempt to include their own DVA in the closeout amount.

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[2] "Application of own credit risk adjustments to derivatives - consultative document", Basel Committee on Banking Supervision, http://www.bis.org/press/p111221.htm and http://www.bis.org/press/p120725b.htm.
[3] We note that we do not consider DVA as a funding benefit nor do we consider the impact of funding costs (FVA) in general in this paper.
[4] Meaning those with credit spreads which are highly correlated to those of the institution.
[5] Although we note that any gain related to DVA would be lost if entering into an equivalent replacement transactions as it would then need to be paid as a CVA charge.

An additional theoretical complexity brought about by the use of bilateral CVA (BCVA) is that it implies that the CVA alone depends on the credit quality of the institution in question. This is because the probability of default of the counterparty must be weighted by the probability that the institution has not previously defaulted. This captures the "first to default" nature of a contract with respect to the default of the institution and counterparty and avoids a double counting. However, it also means that even a pure asset appears to bear the credit risk of both parties which is counterintuitive. However, Brigo and Morini (2011) have shown that in such a case, the dependence on "own default risk" disappears if a risky closeout is assumed.

2. Bilateral CVA

Extending the classic CVA formula bilaterally leads to the following representation (e.g. see Gregory 2009, Brigo et al. 2011):

$$BCVA = CVA + DVA = \int_0^\infty EE(t)[1 - F_I(t)]\, dF_C(t) + \int_0^\infty NEE(t)[1 - F_C(t)]\, dF_I(t), \quad (1)$$

where $EE(t)$ and $NEE(t)$ represent the expected exposure and negative expected exposure (including discounting) and $F_C(t)$ and $F_I(t)$ are the cumulative default probabilities of the counterparty and institution respectively. The above formula assumes that the defaults are independent although this can be readily relaxed (e.g. see Gregory 2009). Putting other potential objections to DVA aside, an issue with the above formula is that an institution's own default probability impacts their CVA. Furthermore, the assumption of independent defaults is a strong one and some model for this dependency should surely be chosen. However, some institutions calculate both CVA and DVA unconditionally (UBCVA) according to:

$$UBCVA = UCVA + UDVA = \int_0^\infty EE(t)\, dF_C(t) + \int_0^\infty NEE(t)\, dF_I(t), \quad (2)$$

This may appears somewhat naïve at first glance as there is a clear "first-to-default" effect in that contracts are terminated at the first default time of either party. However, the results of Brigo and Morini (2011) show that in a unilateral case, UCVA (or UDVA) is the correct formula in the case of a risky closeout assumption. This would tend to suggest that equation (2) is indeed the correct representation of bilateral CVA.

However, according to a recent survey[6], banks are divided on whether to use conditional or unconditional representations (see also Carver 2011). The aim of this paper is therefore to extend the Brigo and Morini unilateral case to consider the "right" approach. Unfortunately, this will be far from trivial and not allow an unambiguous answer. However, we will describe assumptions that will make the UBCVA approximately (but not exactly) valid.

3. Closeout and DVA

[6] See "Reflecting credit and funding adjustments in fair value", Ernst and Young, available from http://www.ey.com/. In this survey, out of the banks using DVA, six state they use a "contingent approach" whilst seven use a non-contingent approach. These approaches respectively correspond to BCVA and UBCVA as defined above.

In deriving formulas for CVA and DVA, a standard assumption is that, in the event of default, the closeout value of transactions (whether positive or negative) will be based on risk-free valuation. This is an approximation that makes quantification more straightforward but the actual payoff is more complex and subtle. Let us consider the situation when a counterparty defaults. Suppose the valuation is negative, say -$900, with a DVA component making it -$800. A risk-free closeout would require the institution to pay $900 and also make an immediate loss of $100. If the DVA can be included in the closeout calculation then the institution pays only $800 and has no jump in their PnL that would otherwise occur (Brigo and Morini [2010]). If instead the institution has a positive valuation of $1,000, of which $900 is risk-free value and $100 is DVA[7] then a risk-free closeout amount is based on $900, leading to a certain loss of $100. On the other hand, a risky closeout allows a claim of $1,000 which matches[8]. Documentation tends to support this approach, for example, under ISDA (2009) protocol, the determination of a closeout amount "*may take into account the creditworthiness of the Determining Party*", which suggests that an institution may consider their own DVA in determining the amount to be settled.

Brigo and Morini [2011] show that the inclusion of DVA in the closeout amount generally leads to a more intuitive theoretical result than a risk-free closeout. These authors illustrate the impact on a zero coupon bond and discuss the special cases of independence and perfect correlation of default times. The zero coupon bond alone (one sided payoff profile) might be quite a limiting simplification, since it naturally neglects one side of the CVA/DVA pair. There are three potential ways in which to extend such an analysis. The first of these is to consider the impact of default correlation (and/or spread volatility and spread correlation) on the results. The second is to look at the recursive nature of this effect (the closeout amount has an impact on the current CVA and DVA ***and*** vice versa). The third and very important point of interest is to calculate the impact on bilateral derivatives exposures.

In order to account for risky closeout in counterparty risk valuation, an institution should quantify the additional gain arising when their counterparty defaults. This comes from two components; the first is an increased claim in the event of a positive future value (of which a recovery will be achieved). The second is a gain resulting from offsetting any amount owed by the DVA. The situation we assume under risky closeout is represented in Figure 1. A positive value leads to a claim on the amount owed, which includes the cost of DVA that would be incurred on a replacement transaction[9]. A negative value requires a settlement of the amount to the counterparty which is offset by the DVA[10].

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[7] This could arise from two outstanding payments where the institution receives $1,900, and pays $1,000. The $100 DVA is coming from this $1,000 payment.

[8] However, there will still be a loss due to the non-recovered amount of the DVA. The amount of CDS protection required to hedge would then be $900 + $100/(1-Recovery), where the second component hedges the DVA loss and is sensitive to the highly uncertain recovery value.

[9] Note that only a recovery fraction of this DVA will be received.

[10] Note that this could turn the amount owed into a claim. This is accounted for in the formulas given below.

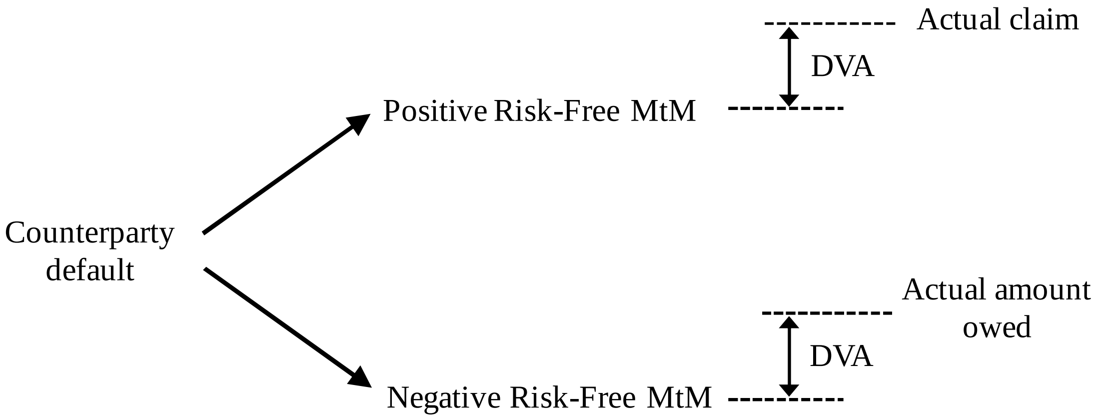

Figure 1. *Illustration of the impact of DVA on the closeout amount when a counterparty defaults.*

An institution also needs to consider the symmetric case which occurs when they themselves default. In this case, the counterparty can increase their valuation in exactly the same way. To the institution, this *increase* in valuation from DVA appears as a *reduction* in valuation by CVA. The four resulting cases are shown in Table 1 (these four cases are incorporated directly into the bilateral CVA formula in equation 8 in Brigo et al. 2011). Having CVA and DVA appear in their own payoff is complex but seemingly unavoidable. Indeed, similar effects occur in cases such as the exercise of physically settled options where the CVA and DVA of the underlying impact the exercise boundary[11].

Table 1. *Comparison of payoffs using risk-free and risky closeout. Risk-free closeout is defined in the usual way via the risk-free mark-to-market (denoted MtM). In risky closeout, when the counterparty defaults, the institution increases the valuation by their own DVA (which is negative by convention). When the institution themselves defaults, the counterparty can reduce the valuation by their DVA (which from the institution's point of view is their CVA).* R_C *and* R_I *represent recovery values of the counterparty and institution respectively.*

	Risk-free closeout		Risky closeout	
	Positive Exposure	Negative Exposure	Positive Exposure	Negative Exposure
Counterparty defaults	$R_C \times$ *Max*(MtM,0)	*Min*(MtM,0)	$R_C \times$ *Max*(MtM - DVA,0)	*Min*(MtM - DVA,0)
Institution defaults	*Max*(MtM,0)	$R_I \times$ *Min*(MtM,0)	*Max*(MtM - CVA,0)	$R_I \times$ *Min*(MtM - CVA,0)

We note that there are some potential objections to the above stylised assumptions regarding closeout amounts which will be discussed at the end of this article. However, we will first show that under the assumptions described above and represented by Figure 1 that the strong "first to default" effect of bilateral CVA valuation is largely removed when assuming risky closeout. However, in contrast to previous research, we will also show that, even then, risky closeout is not a perfectly clean theoretical solution in that aspects such as default correlation are still important.

[11] See, for example, Arvanitis and Gregory [2001].

4. Simple example

A good intuition of bilateral CVA and closeout interdependence is provided by analysing a simple case of cashflows in opposite directions. The logic would be the same regardless of the sizes of those cashflows[12], so to simplify the exposition we assume them to be equal. Assume an institution pays a unit cashflow at time T_1 and receives a unit cashflow at a later time T_2 (Figure 2) We assume that both the institution (*I*) and their counterparty (*C*) can default and have associated fixed hazard rates of h_I and h_C respectively. Percentage recovery rates are given by R_I and R_C and interest rates are assumed to be zero. The exposure based on risk-free closeout is zero until T_1 and +1 from T_1 to T_2. The fact that the above case represents only positive exposure is not a concern due to the inherent symmetry of the problem (although we deal with the more general case below). The aim now is to compute the formula for the CVA.

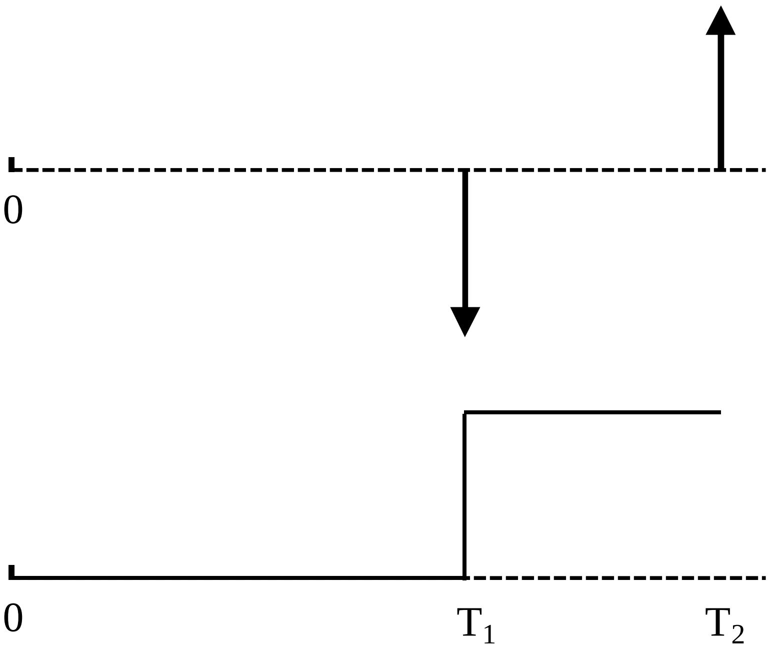

Figure 2. *Illustration of the simple example showing the cashflows (top) and exposure (bottom).*

Note that the representation below, for ease of exposition, assumes independence of defaults but the more general case is an easy extension, for example we can represent the hazard rates under conditional independence as in some factor model. We define $F(T_1, T_2)$ as the default probability between dates T_1 and T_2 and $S(T_1, T_2)$ as the associated survival probability. We denote the first to default probability and associated survival functions as $F^1(.)$ and $S^1(.)$ respectively. With a standard closeout based on the risk-free value of the claim, the CVA at time zero, which intuitively should reflect the fact that if the counterparty defaults first in the interval [T_1, T_2] then the institution makes a loss due to not receiving the final cashflow, can be written as:

$$(1 - R_C) h_C \int_{T_1}^{T_2} \exp(-(h_c + h_I)s) ds = (1 - R_C) \frac{h_c}{h_c + h_I} F^1(T_1, T_2), \quad (3)$$

[12] Unless the case degenerates due to very significant difference in cashflow sizes.

where $F^1(T_1, T_2) = \exp(-(h_c + h_I)T_1) - \exp(-(h_c + h_I)T_2)$ is the first to default probability within the interval $[T_1, T_2]$. The ratio $h_c/(h_c + h_I)$ gives the probability that the counterparty is the first to default. This formula has a dependence on the institution's own default probability via the first to default probability. As the institution's default probability increases, the CVA tends to zero.[13]

Let us now look at the impact of "risky closeout" (including DVA) on the above calculation. If the institution defaults, the counterparty will include DVA (or CVA from the institution's point of view). We have to therefore consider two additional components corresponding to the two different time periods[14].

i) Institution defaults first in the period [0, T_1].

Here the counterparty will claim their DVA benefit (which is the institution's CVA) but will receive only a recovery fraction of it. This requires an addition term of:

$$R_I h_I \int_0^{T_1} CVA_{\tau_I=s}(s)\exp(-(h_c + h_I)s)ds, \tag{4}$$

which evaluates the CVA component at the default time of the institution. Since the institution has defaulted, its hazard rate will drop to zero[15] and the CVA will become $CVA_{\tau_I=s}(s) = (1 - R_C)[\exp(-h_c(T_1 - s)) - \exp(-h_c(T_2 - s))]$. Substituting this into the above and integrating again, we obtain:

$$R_I(1 - R_C)F_C(T_1, T_2)F_I(0, T_1). \tag{5}$$

The intuition behind this is that if the institution defaults before T_1 and then the counterparty defaults in the interval [T_1, T_2] then the counterparty will claim their DVA on the remaining cashflows and the institution (because they are in default) will pay only a recovery fraction of this. Another way to look at this is to consider how much it will cost the counterparty to replace the transaction in case of the institution defaulting prior to T_1. A party providing the replacement transaction will have to assess the probability of the counterparty default in the interval $[T_1, T_2]$ and will incorporate this in the price.

ii) Institution defaults first in the period [T_1, T_2].

Here the counterparty will subtract their own DVA from the unit payment they are obliged to make. Since they owe the institution then there is no recovery value as in the previous case. This gives an additional term of:

$$h_I \int_{T_1}^{T_2} CVA_{\tau_I=s}(s)\exp(-(h_c + h_I)s)ds,$$

The CVA at this point will be $CVA_{\tau_I=s}(s) = (1 - R_C)[1 - \exp(-h_c(T_2 - s))]$. Again evaluating the integral gives:

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[13] We note that if we consider the risk-free closeout, this case is not different from the zero bond case mentioned above, apart from the reduction in the relevant time frame.

[14] Note that we do not need to consider the default of counterparty because they cannot be a creditor in this example.

[15] This arises since we assume the replacement trade will be with a risk-free counterparty.

$$(1 - R_C)\left[\frac{h_I}{h_C + h_I} F^1(T_1, T_2) - S_C(0, T_2) F_I(T_1, T_2)\right] \tag{6}$$

The probability in the brackets gives the probability that the institution defaults in the interval [T_1, T_2] and the counterparty defaults second but before T_2. The CVA with risky closeout is found by adding the terms in equations (3, 5 and 6) above, giving:

$$UCVA - (1 - R_I)(1 - R_C) F_C(T_1, T_2) F_I(0, T_1), \tag{7}$$

where the unilateral CVA is given by $UCVA = (1 - R_C) F_C(T_1, T_2)$. The second term is a correction due to the fact that, in the event of the institution's own default, the counterparty may claim a recovery fraction of their DVA benefit. If $T_1 = 0$, or equivalently, when the institution has no liability then we obtain the result of Brigo and Morini (2011) corresponding to the UCVA with no sensitivity to the institution's own hazard rate. However, in the bilateral case, neither CVA nor UCVA is the correct solution to the problem and there is an adjustment term. In Figure 3, we compare the different closeout assumptions for this simple example showing CVA, UCVA and the true risky closeout result of equation (7). In this example, the actual result is somewhere in between CVA and UCVA.

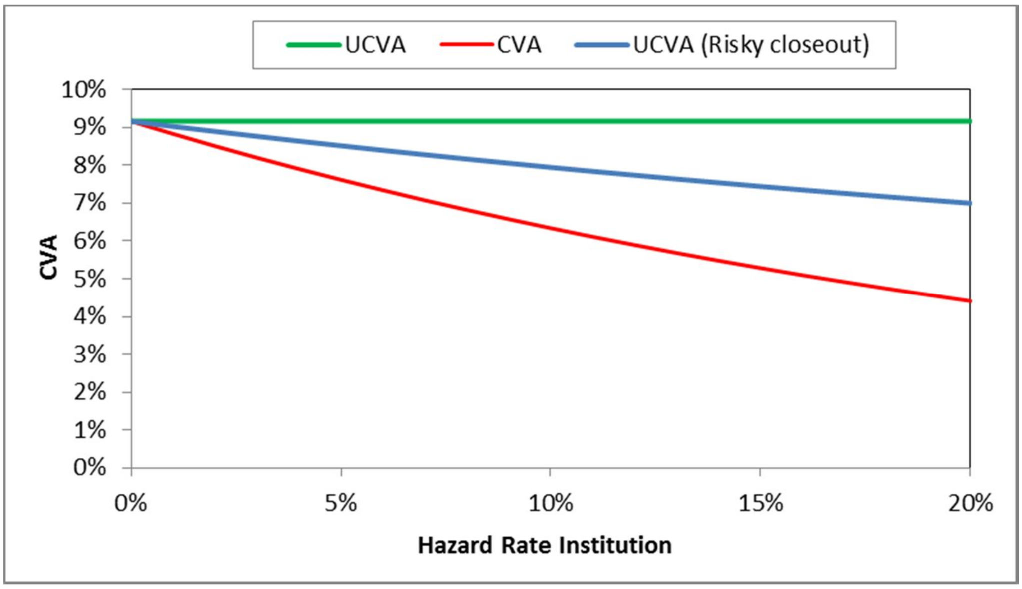


Figure 3. *CVA for the simple two cashflow example with T_1=2.5 years and T_2 = 5 years, computed with both risk-free and risky closeout as a function of the hazard rate of the institution. The counterparty hazard rate is 8.33% and recovery rates are 40%.*

We have seen that in the general bilateral case, risky closeout assumptions do not lead to an obvious simple CVA formula as they do in the unilateral case of Brigo and Morini (2011). However, the above example was rather extreme as only one party had a DVA component. Furthermore, we have not yet considered the impact of other aspects such as default correlation. We will look at the more general case below.

5. Actual example

We now take an example with bilateral exposures based on the exposure profiles shown in Figure 4 which are representative of a typical swap[16]. In this portfolio the negative expected exposure (which drives the DVA) is greater in absolute terms than the expected exposure (which drives the CVA). If we assume that h_C=8.33% and h_I = 4.17% so that the counterparty is more risky than the institution then this gives an case where the CVA and DVA are approximately equal and opposite and the BCVA is close to zero (Table 2). We also show the UBCVA results show a similar behaviour but also give rise to a materially different valuation.

Table 2. *Unilateral and bilateral CVA values and the corresponding unconditional values for the swap portfolio assuming independence between default events.*

Conditional		Unconditional	
CVA	149,800	UCVA	162,407
DVA	-140,213	UDVA	-165,179
BCVA	9,587	UBCVA	-2,772

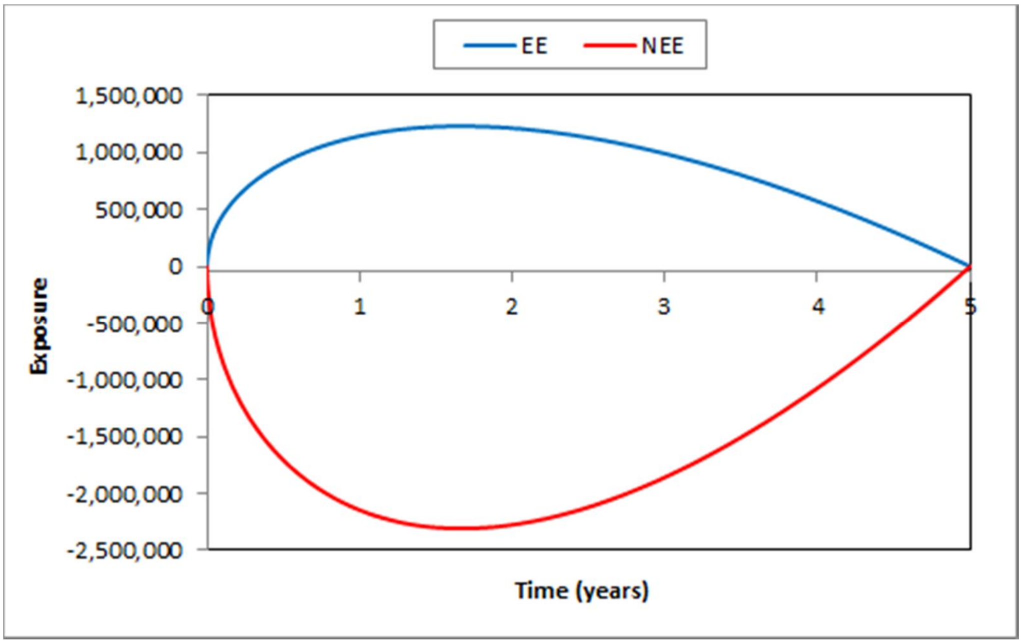


Figure 4. *Expected exposure (EE) and negative expected exposure (NEE) profiles used for the bilateral calculations.*

In order to introduce dependence between the default times of the institution and counterparty, we use a simple and well-known Gaussian copula approach. Another aspect to consider is that the CVA (or equivalently DVA) defined at the time of closeout should naturally include the value of any future closeout adjustments (on the replacement transaction) which leads to a recursive problem. We solve this by simply recalculating the above integrals numerically and iteratively solving until a convergence is reached. More details can be found in Gregory and German (2012).

The impact of correlation on the BCVA (Figure 5) shows a strong effect with BCVA increasing towards the unilateral value as the correlation increases to 100%. This is due to the

[16] These profiles are generated via $EE(t) = -0.25(T-t)\sqrt{t}\Phi(-0.25) + (T-t)\sqrt{t}\varphi(-0.25)$ and $NEE(t) = -0.25(T-t)\sqrt{t}\Phi(0.25) - (T-t)\sqrt{t}\varphi(0.25)$ which arises via the assumption that the future value at each date t is normally distributed with mean $-0.25 \times (T-t)\sqrt{t}$ and standard deviation $(T-t)\sqrt{t}$.

aforementioned comonotonic feature where the most risky name is certain to default first and therefore the DVA benefit is lost. The “first to default” impact on BCVA is clearly very significant. On the other hand, the results of the BCVA with a risky closeout (including the impact of DVA and CVA and the recursive effect) show that default correlation now has a much smaller impact on the BCVA. This is due to the fact that the institution can benefit from their DVA even in the event that the counterparty defaults first.

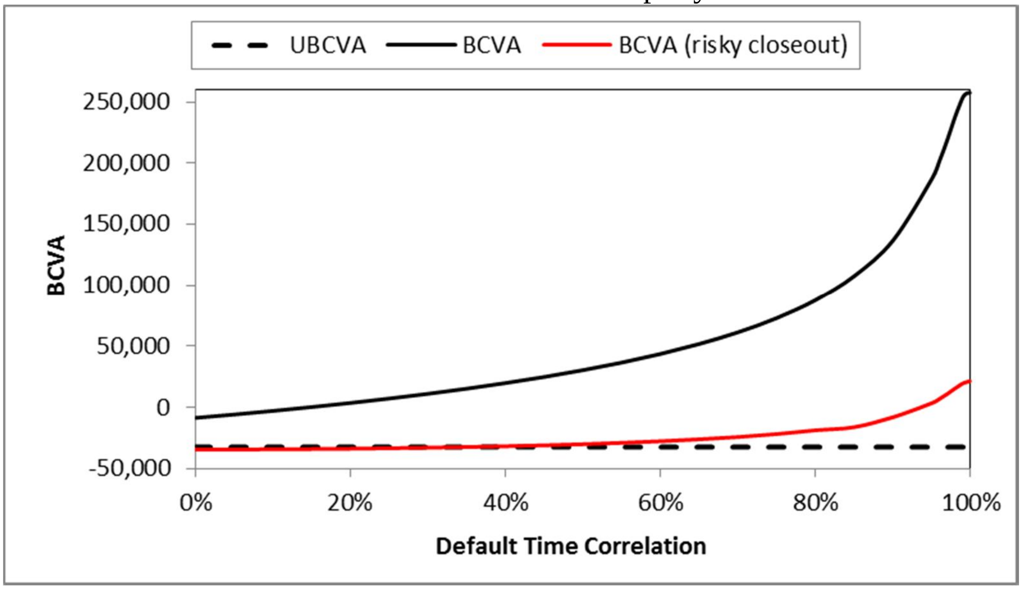
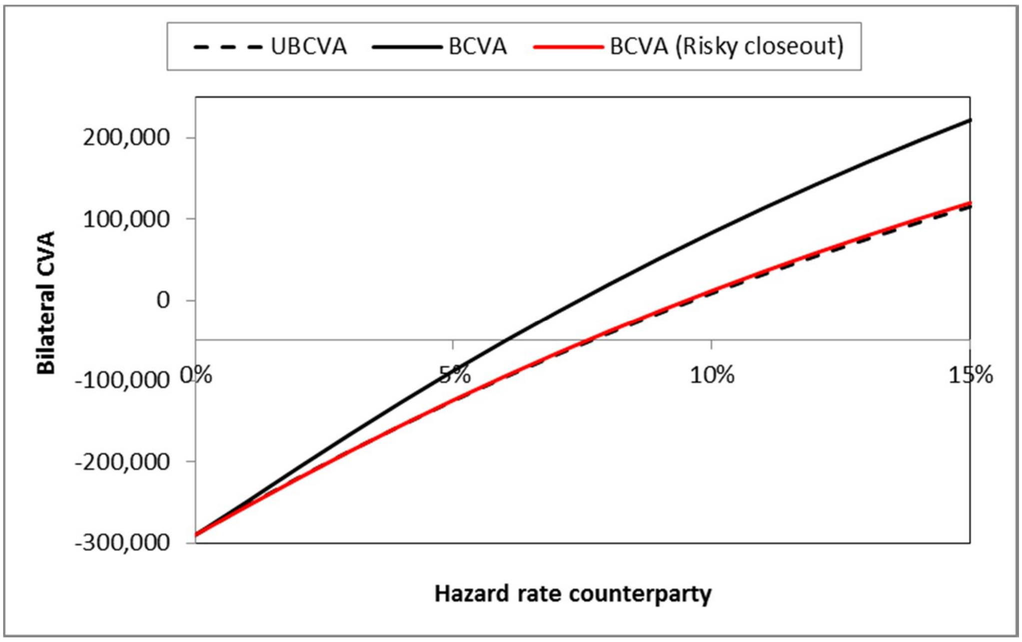


Figure 5. *Illustration of the impact of risky closeout on the BCVA for the bilateral example as a function of the correlation between the default of the institution and their counterparty. Also shown is the approximation arising from using the unconditional BCVA (UBCVA).*

Interestingly, in this more general case, the UBCVA approach gives close agreement with the case of risky closeout especially for low correlation values. We test this over a wider range of situations and Figure 6 shows the same quantities as a function of the hazard rate of the counterparty and institution for a fixed default time correlation of 50%. Whilst we know from the simple result given in equation (7) that UBCVA will not always give the correct risky closeout valuation, in more realistic cases it appears to be in very close agreement.



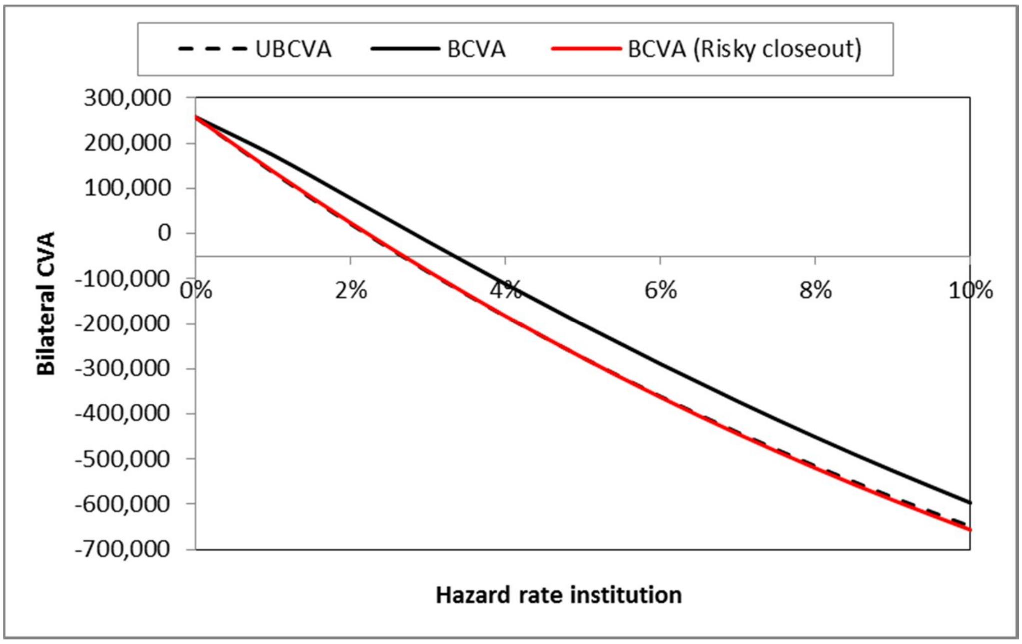


Figure 6. *Illustration of the impact of risky closeout on the BCVA for the bilateral example as a function of the hazard rate of the counterparty (top) and institution (bottom). Also shown is the approximation arising from using the unconditional BCVA (UBCVA). A fixed default time correlation of 50% is used.*

6. Conclusion

We have examined the pricing of bilateral counterparty risk using risky closeout assumptions where a surviving party would be able to include their DVA in the amount paid or claimed from their defaulted counterparty. Risky closeout tends to cancel out some of the complicated features created by the use of DVA, in particular the strong impact of correlation between defaults. It seems unlikely that, given the complexity of CVA computation, any institution would attempt to properly reflect risky closeout assumptions, especially since doing so requires a recursive calculation. It is therefore partially reassuring that the UBCVA formula gives a very close result to the true risky closeout case in the example considered above[17]. Our results suggest that, in the absence of a more complex calculation, UBCVA should be used rather than BCVA. Since, as mentioned earlier, the market appears rather equally divided between these choices, this is an important conclusion.

However, unfortunately the bilateral case is not clear cut as the previous unilateral case considered by Brigo and Morini (2011). In extreme cases, UBCVA may not be a particularly good approximation to the actual case (as seen in Figure 3)[18], especially when an institution's own default probability is high. In certain cases therefore, it appears important to take into consideration the dependency between default and the recursive nature of the bilateral CVA payoff.

An added problem is that the precise assumptions we have made for risky closeout could also be questioned[19]. An intuitive criticism could be the lack of recognition of the CVA of the replacement transaction, i.e. the implicit assumption that the replacement counterparty is risk-free. Or one might consider a dealer market with homogeneous credit quality and symmetric exposures. Here, CVA and DVA are reduced by the use of collateral and should in any case cancel, so that the correct replacement cost (ignoring transaction costs) would simply be the risk-free amount. It remains to be seen what the implication of using other assumptions would be but it is unlikely that they will simplify the complex problem of default dependency and closeout assumptions in the pricing of bilateral counterparty risk. Furthermore, the possible inclusion of potential funding costs in closeout assumptions, not considered here, will make the problem even more complex.

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[17] We have verified that this is also true for other cases, including actual portfolios.

[18] An obvious practical examples where the exposure flips dramatically in sign, such as in the simple example, would be a portfolio containing a large cross-currency swap which at maturity may result in a very large cash payment from a strong FX move.

[19] Indeed, risky closeout has not always been observed in practice. For, example in the Peregrine Fixed Income Limited v Robinson Department Store plc case (e.g. see Parker and McGarry 2009) although we note that the most recent ISDA documentation support risky closeout more than previous versions.

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